FAIR LANDS

QUICK START

EXCELLENCE — EQUITY

FAIR LANDS™

Acquisition of the concept of equivalent fractions—the key to understanding and working with fractions

Acquisition of the concept of fair trades fundamental to success with time, money, and measurement

Mastery of subtraction of whole numbers with under-standing for ALL students

Mastery of addition of whole numbers with under-standing for ALL students

EXCELLENCE

EQUITY

CONTENTS

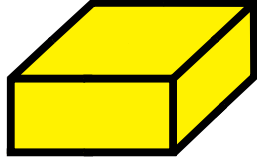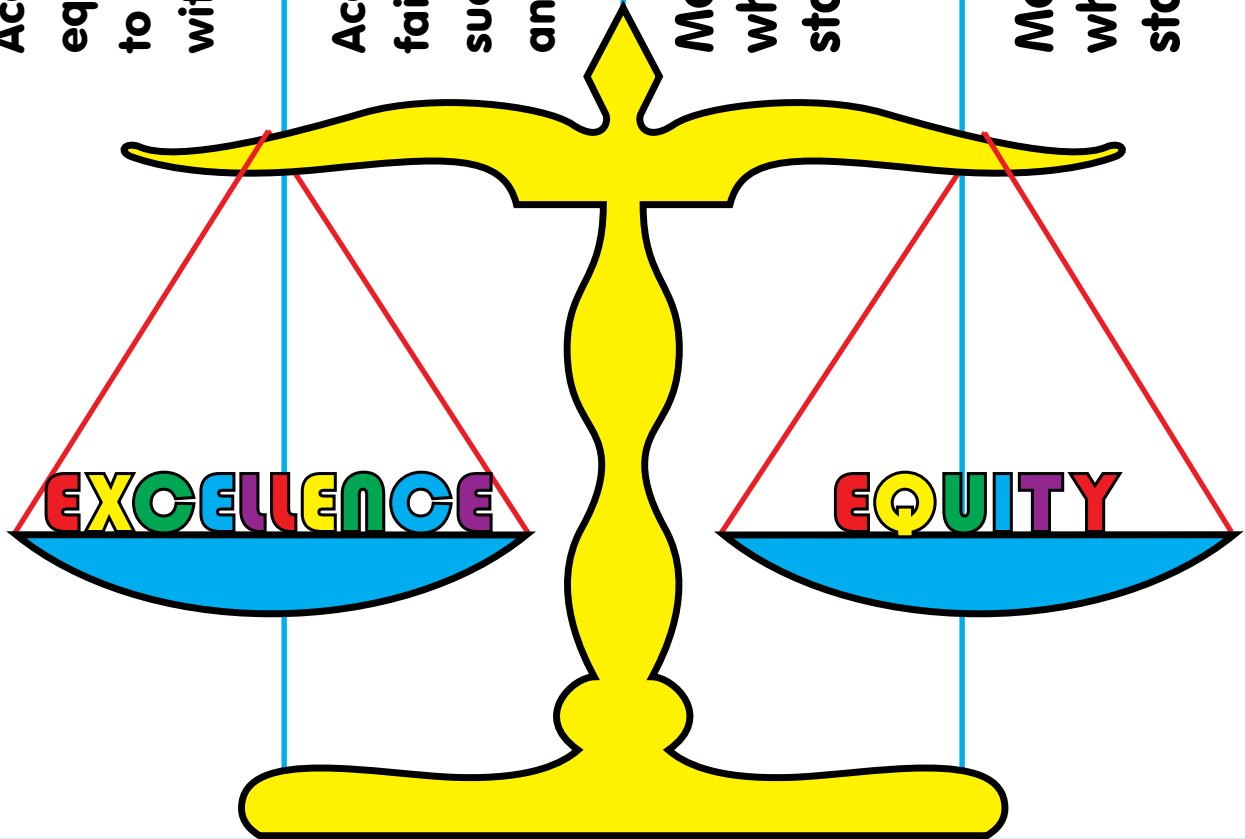*Fair Lands™ QUICK START*

Fair Lands™

**F**air Lands is a trading center for fair trades, particularly ones based on numbers like five pennies for a nickel, 60 minutes for an hour, 12 inches for a foot, and two halves for a whole. Since the numbers are backed by law or convention, the items traded are said to be in a land, and the land is named after the number that prescribes the trade. So for the trades mentioned, pennies and nickels are in Five Land because five pennies make a nickel; minutes and hours are in Sixty Land because 60 minutes make an hour; inches and feet are in Twelve Land because 12 inches make a foot; and halves and wholes are in Two Land because two halves make a whole.

Fair Lands Blocks

This booklet is about two exceptional lands—ones made of specially designed blocks. One is called Two Land because every *two* identical blocks make the next bigger block. The other is called Three Land because every *three* identical blocks make the next bigger block. How they are made is shown on the last two pages in this guide.
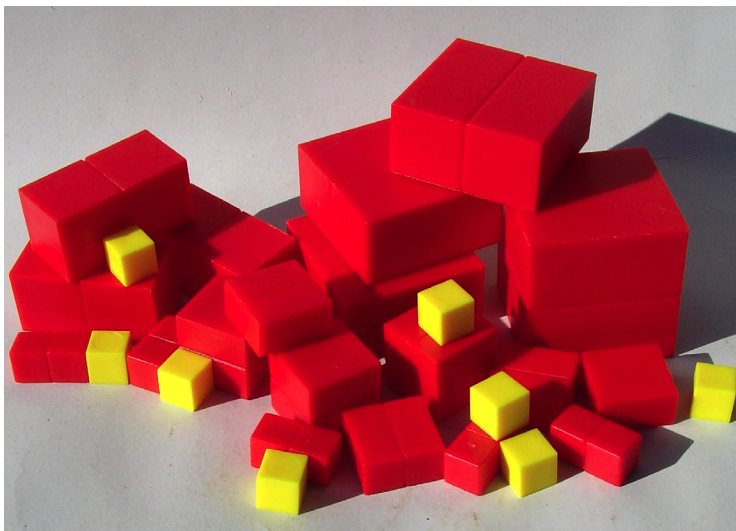
Two Land Blocks
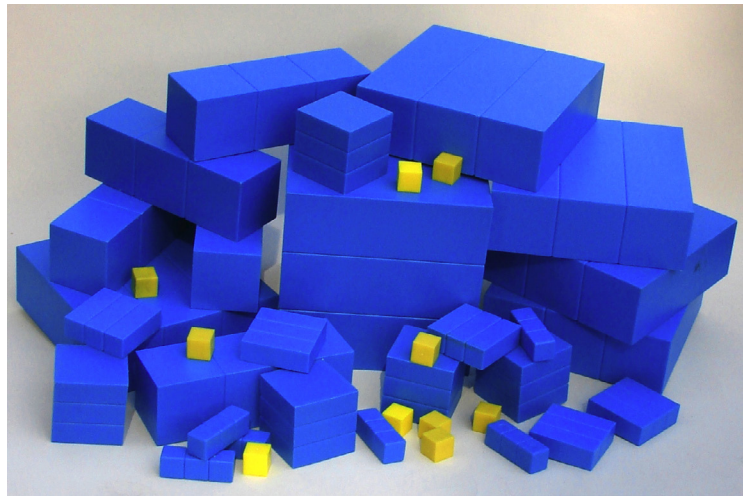
Three Land Blocks

These lands differ from most lands by the strictness of their laws. In these lands, if a trade can be made, it must be made. In Two Land, every two blocks the same must be traded for one of the next bigger block, and in Three Land, every three blocks the same must be traded for one of the next bigger block. In contrast, seven pennies in Five Land may be traded for a nickel and two pennies; 75 minutes in Sixty Land may be traded for an hour and 15 minutes; 30 inches in Twelve Land may be traded for 2 feet, 6 inches, and 3/2 may be traded for 1-1/2.

Why Two Land and Three Land?

Adding and subtracting in any place value numeration system are making and recording fair trades. For elementary school students to learn this with understanding, they must perform these operations in more than base 10. Then what stays the same when the base numbers change stands out, and what stands out is that adding and subtracting are just about making and recording fair trades.

Two Land (base 2) and Three Land (base 3) were chosen as alternate bases because …

•Students don't need to know the addition and subtraction facts to add and subtract with just 0, 1, and 2.

•The smaller the base, the more practice making and recording fair trades.

•Place value in the systems could be represented with blocks out to seven places without the blocks getting too big.

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What Is the Objective?

The goal in adding and subtracting in Two Land and Three Land is for elementary school students to learn to make and record fair trades. It is not to acquire expertise in base 2 and base 3 arithmetic, though this may occur. It is to understand base 10 numeration and learn addition and subtraction as ways of thinking rather than rote procedures.

Getting Used to the Fair Lands Blocks

Students need to get past their natural excitement at first handling the blocks.

Materials

•Fair Lands blocks (both lands)

Lesson

Let students play and build with the blocks, perhaps for days and even weeks. When they can answer the following questions through their own investigations, they're ready to work with the blocks.

•Which "land" is Two Land? (The red blocks.) Three Land? (The blue blocks.)

•Why are the red blocks called Two Land? (Because two of any block make one of the next bigger block.)

•Why are the blue blocks called Three Land? (Because three of any block make one of the next bigger block.)

•Why isn't the smallest block red or blue? (It's the unit or "1 block" for both lands, so it had to be another color and the same in both lands.)

Names of the Fair Lands Blocks

Students need to learn the names of the blocks and their placement on the Fair Lands activity boards. Click on the pictures of the activity boards for copies you can print.

Materials

•Fair Lands blocks (both lands)

•Fair Lands 4-column activity board and 7-column activity board
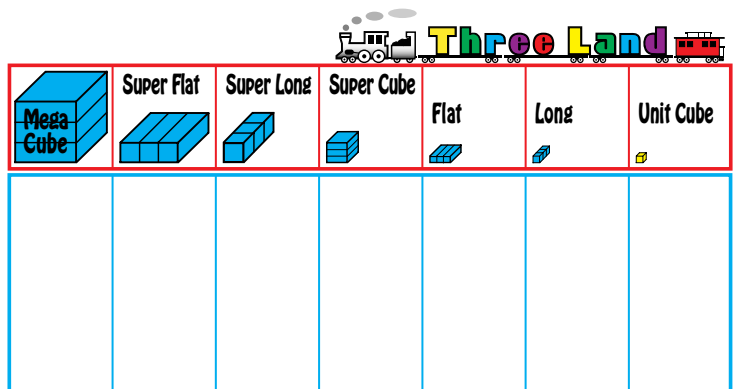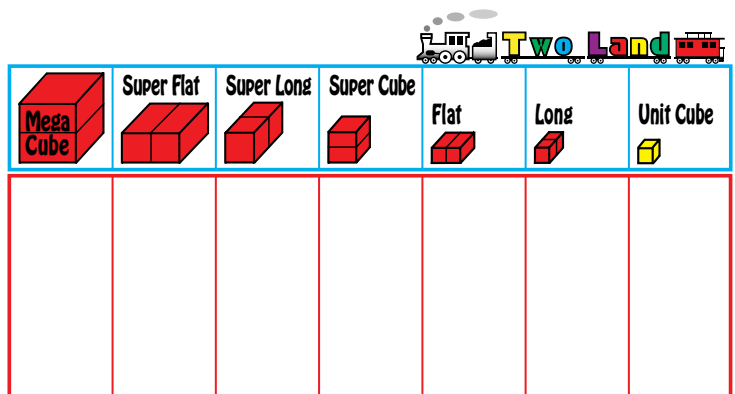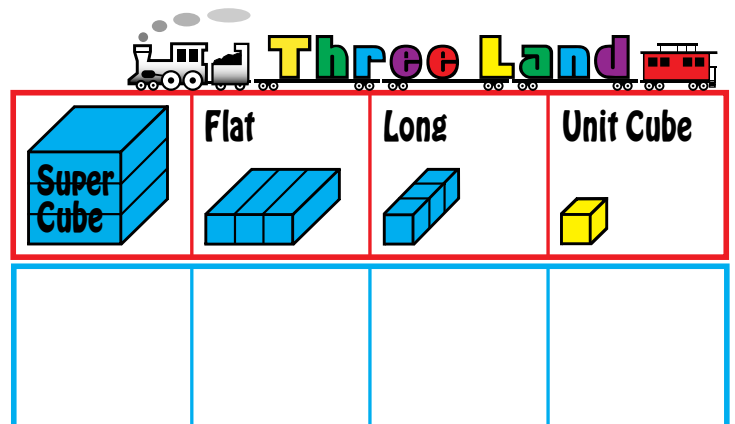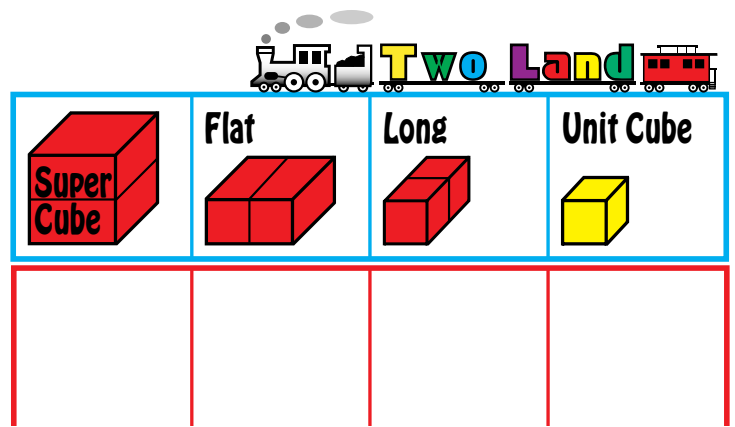
Lesson

•Match the Fair Lands blocks with the illustrations of the blocks on the Fair Lands activity boards.

•Classify the blocks by placing them on the activity boards.

Readiness Activity to Add with the Fair Lands Blocks

To add with the Fair Lands blocks, elementary school students need to learn when and how to trade "up": little blocks for big ones. This is taught by having them play the Fair Trades UP penny game where they toss a coin and add units to their activity boards one or two at a time depending on the toss. Afterwards, they "legalize" their boards if they end up with more of the same block than is allowed.

Fair Trades UP Penny Game in Two Land and Three Land

Materials

The materials for the game are listed with the instructions for it.

Lesson

Play the Fair Trades UP penny game in Two Land and Three Land with the Fair Lands blocks. Then play it in at least one other land with counters. If Ten Land blocks are available, play it in Ten Land, too, but just a little because the bigger the land, the less practice on trading.

In playing the game, stress KISS, an acronym for "Keep it safe and simple," meaning keep activity boards safe and trades simple as explained in the instructions for the game. This ensures working right-to-left and column-by-column like in arithmetic.

Two Land Example

Imagine a Fair Trades Up penny game in Two Land between two players, Lion and Tiger. They've been playing for a while, and Lion has one Flat, one Long, and one Unit Cube on his activity board. Let's see if he wins on his next coin toss.

S. Cube	Flat	Long	Unit C.
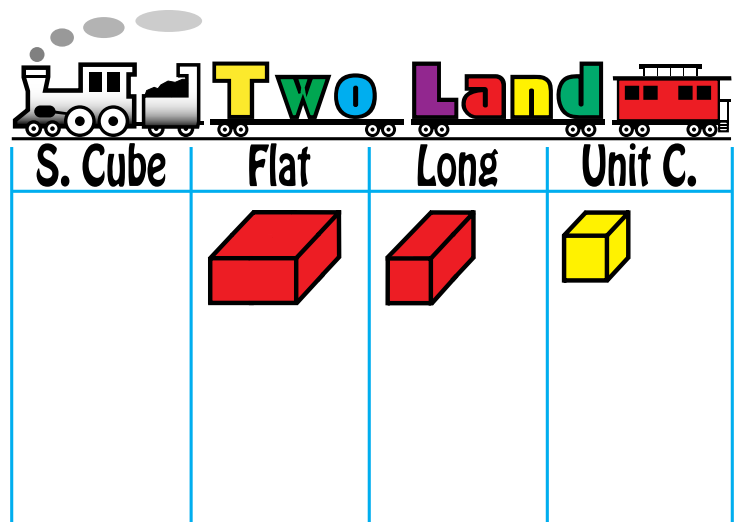
Lion tosses a tail, so he puts two Unit Cubes on his board.

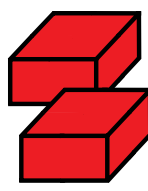| S. Cube	Flat	Long	Unit C.
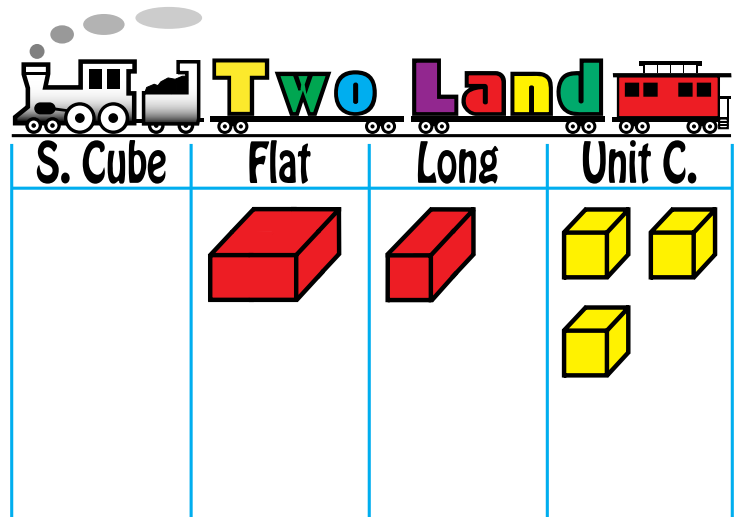
Too many Unit Cubes! So keeping it "simple," he trades two Unit Cubes for a Long (not two Unit Cubes and the Long for a Flat).

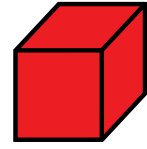| S. Cube	Flat	Long	Unit C.
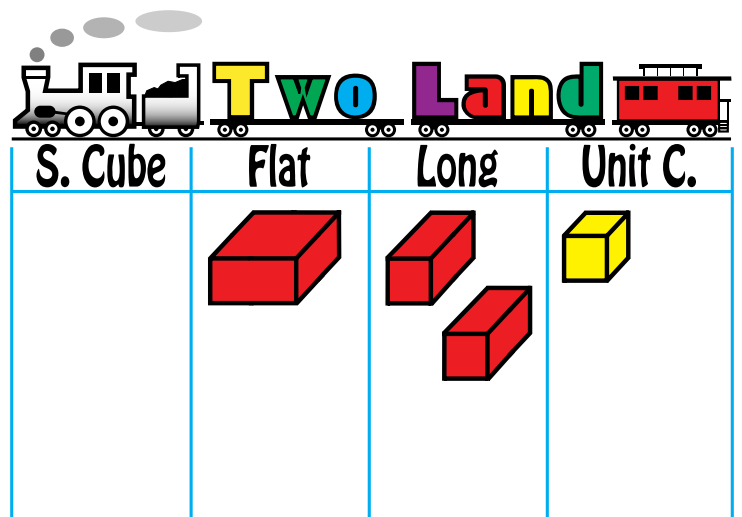
Too many Longs! So keeping it "simple," he trades two Longs for a Flat (not the Longs and Flat for a Super Cube).

3

S. Cube	Flat	Long	Unit C.
	(red flats)		(yellow cube)

Too many Flats! So he trades two Flats for a Super Cube.

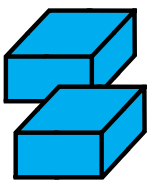| S. Cube	Flat	Long	Unit C.
(red cube)			(yellow cube)

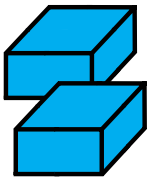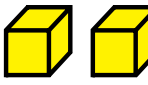Finally, his board is safe, and he wins because he got a Super Cube before Tiger.

Three Land Example

Imagine Lion and Tiger playing another Fair Trades UP penny game, except in Three Land. They've been playing for a while and Tiger has two Flats, one Long, and one Unit Cube on his activity board. Let's see if he wins.

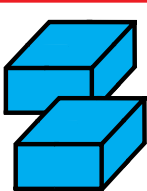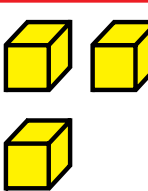| S. Cube	Flat	Long	Unit C.
	(blue flats)	(blue long)	(yellow cube)

It is Tiger's turn and he tosses a head, so he puts a Unit Cube on his board. The board is safe, so his turn is over.

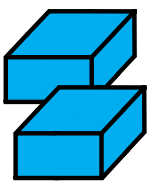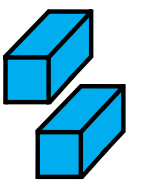| S. Cube	Flat	Long	Unit C.
	(blue flats)	(blue long)	(yellow cubes)

It is Tiger's turn again and he tosses a head, so he puts a Unit Cube on his board.

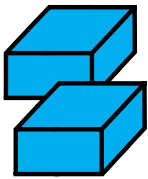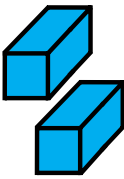| S. Cube	Flat	Long	Unit C.
	(blue flats)	(blue long)	(three yellow cubes)

Too many Unit Cubes! So he trades three Unit Cubes for a Long. His board is now safe, so his turn is over.

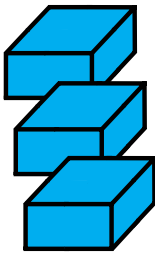| S. Cube	Flat	Long	Unit C.
	(blue flats)	(blue longs)	

It is Tiger's turn again and he tosses a tail, so he puts two Unit Cubes on his board. The board is safe, so his turn is over.
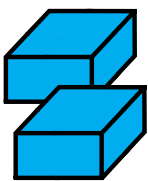
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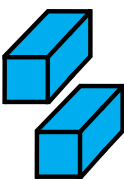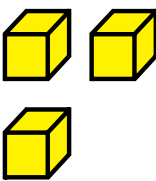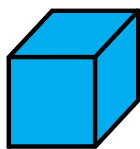It ias Tiger's turn again and he tosses a head, so he puts a Unit Cube on his board.

Too many Flats! So he trades three Flats for a Super Cube.
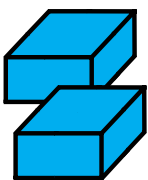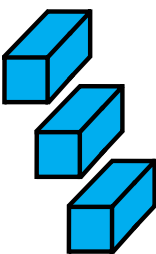

Too many Unit Cubes! So keeping it "simple," he trades three Unit Cubes for a Long (not everything for a Super Cube).

Tiger wins because he beat Lion to a Super Cube.

Readiness Activity to Subtract with the Fair Lands Blocks

To subtract with the Fair Lands blocks, elementary school students need to learn when and how to trade "down": big blocks for little ones. This is taught by having them play the Fair Trades Down penny game—the opposite of the trading up game. Instead of adding Unit Cubes to their activity boards after a coin toss, they take them off one or two at a time dependiing on the toss.

Too many Longs! So keeping it "simple," he trades three Longs for a Flat (still not everything for a Super Cube).

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Fair Trades DOWN Penny Game in Two Land and Three Land

Materials

The materials for the game are listed with the instructions for it.

Lessons

Play the Fair Trades DOWN penny game in Two Land and Three Land with the Fair Lands blocks. Then play it in at least one other land with counters. If Ten Land blocks are available, play it in Ten Land, too, but just a little because the bigger the land, the less practice on trading.

In playing the game, stress keeping trades "simple": always trading the smallest and fewest blocks possible. This ensures working right-to-left and column-by-column like in subtracting. Unlike in the trading up game, keeping one's activity board safe is not an issue. In trading down to have enough Unit Cubes to satisfy a coin toss, the board is made safe the moment the required number of Unit Cubes are removed from it.

Two Land Example

Imagine a Fair Trades DOWN penny game in Two Land between Lion and Tiger. When they started, each of them had one of each block
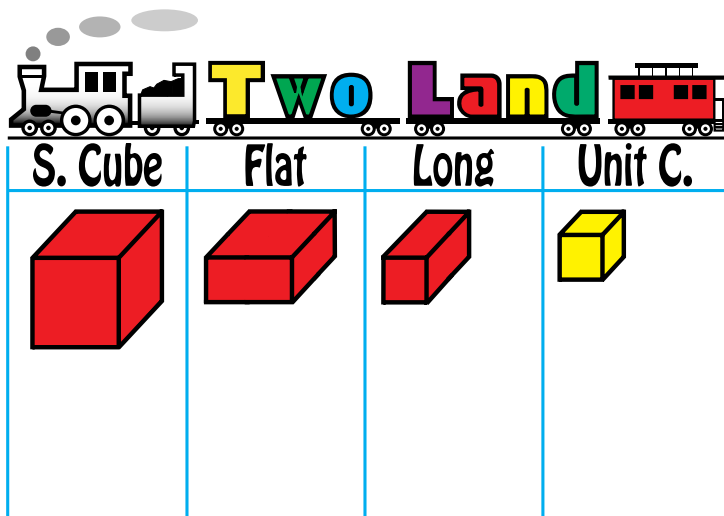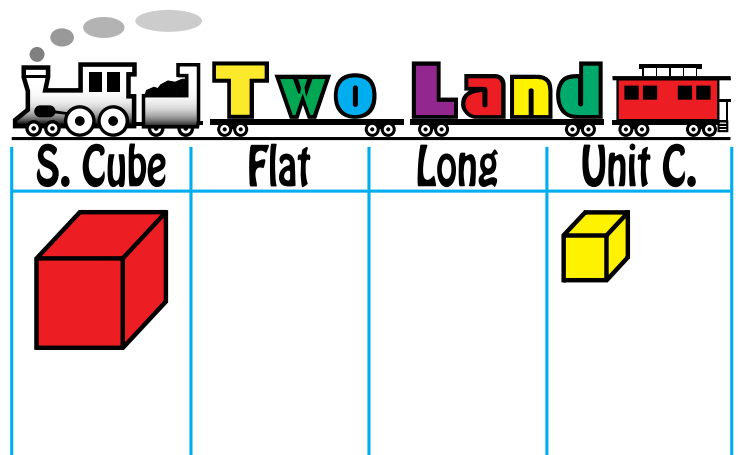

Now that they've played a while, Lion has one Super Cube and one Long on his board. Let's see if he wins.
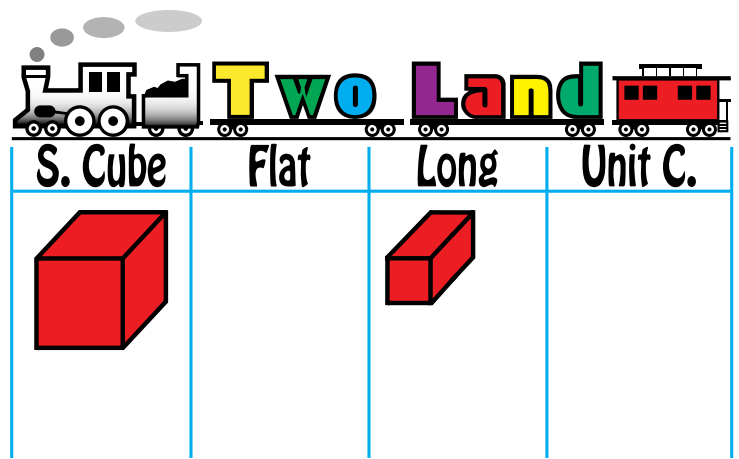
It is Lion's turn and he tosses a head, so he has to take one Unit Cube off his board, but there isn't one. So he trades the Long for two Unit Cubes. Then he takes one Unit Cube off his board.
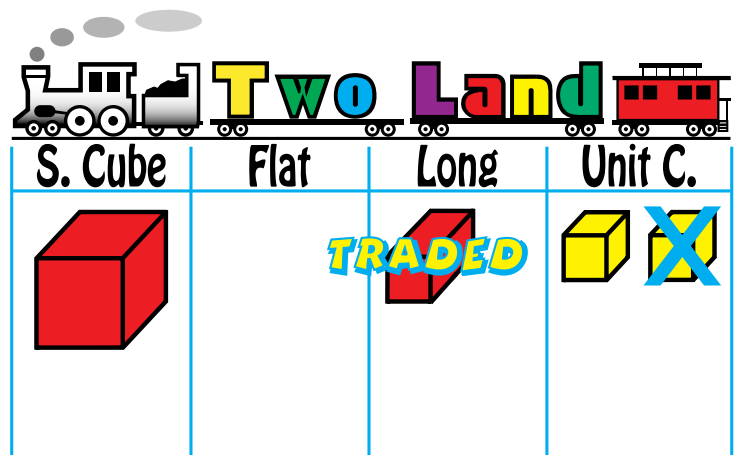
One Super Cube and one Unit Cube are left.

It is Lion's turn again and he tosses a tail, so he has to take two Unit Cubes off his board. Since there is only one, he trades the Super Cube for two Flats. Then he trades one of the Flats for two Longs. Then he trades one of the Longs for two Unit Cubes. Finally, he takes two Unit Cubes off his board.

One Flat, one Long, and one Unit Cube are left.

It is Lion's turn again and he tosses a tail, so he has to take two Unit Cubes off his board. Since there is only one, he trades the Long for two Unit Cubes. Then he takes two Unit Cubes off his board.

One Flat and one Unit Cube are left.

It is Lion's turn again and he tosses a head, so he takes the Unit Cube off his board.

One Flat is left.

It is Lion's turn again and he tosses a tail, so he has to take two Unit Cubes off his board. Since there aren't any, he trades the Flat for two Longs. Then he trades one of the Longs for two Unit Cubes. Then he takes two Unit Cubes off his board.

One Long is left.

It is Lion's turn again and he tosses a head, so he has to take one Unit Cube off his board, but there isn't one. So he trades the Long for two Unit Cubes. Then he takes one Unit Cube off his board.

S. Cube	Flat	Long	Unit C.
		TRADED	

One Unit Cube is left.

S. Cube	Flat	Long	Unit C.

Lion loses! Tiger cleared his board on his turn.

Three Land Example

Imagine another Fair Trades DOWN penny game between Lion and Tiger, this time in Three Land. As before, when they started, each of them had one of each block on their activity board. NOTE: *To speed up the game, they made a head a two and a tail a three.*

S. Cube	Flat	Long	Unit C.

Now that they've played a while, Tiger has two Flats, one Long, and one Unit Cube left on his board. Let's see if he wins.

S. Cube	Flat	Long	Unit C.

It is Tiger's turn and he tosses a head, so he has to take two Unit Cubes off his board. Since there is only one, he trades the Long for three Unit Cubes. Then he takes two Unit Cubes off his board.

S. Cube	Flat	Long	Unit C.
		TRADED	

Two Flats and two Unit Cubes are left.

S. Cube	Flat	Long	Unit C.

It is Tiger's turn again and he tosses a tail, so he has to take three Unit Cubes off his board. Since there are only two, he trades a Flat for three Longs. Then he trades one of the Longs for three Unit Cubes. Then he takes three Unit Cubes off his board.

S. Cube	Flat	Long	Unit C.

One Flat, two Longs, and two Unit Cubes are left.

S. Cube	Flat	Long	Unit C.

Skipping forward, in Tiger's next four turns, he tosses a head, then a tail, then a head, and then a tail. Two Longs and one Unit Cube are left on his board.

S. Cube	Flat	Long	Unit C.

It is Tiger's turn again and he tosses a head, so he has to take two Unit Cubes off his board. Since there is only one, he trades a Long for three Unit Cubes. Then he takes two Unit Cubes off his board.

One Long and two Unit Cubes are left.

| S. Cube	Flat	Long	Unit C.

It is Tiger's turn again and he tosses a tail, so he has to take three Unit Cubes off his board. Since there are only two, he trades the Long for three Unit Cubes. Then he takes three Unit Cubes off his board.

S. Cube	Flat	Long	Unit C.

Two Unit Cubes are left.

S. Cube	Flat	Long	Unit C.

It is Tiger's turn again and he tosses a head, so he takes two Unit Cubes off his board.

S. Cube	Flat	Long	Unit C.
			☒ ☒

Tiger wins again because he cleared his board before Lion.

Preparing to Add and Subtract with the Fair Lands Blocks

After the two penny games, elementary school students need to learn how to represent Two Land and Three Land numerals on a Fair Lands activity board with the Fair Lands blocks. This is taught by having them place the blocks on a board in response to "show me" directives for numerals like $1,010_2$ and $1,100,101_2$ for Two Land and $1,202_3$ and $2,001,021_3$ for Three Land. (The subscripts tell the land.) Then do the reverse: have them write the numerals for blocks already on a board.

About the subscripts, it's important that children become accustomed to them because of the huge use that's made of them in higher math. They may look scary and super abstract, but they're no big deal to children, even young ones. They're nothing but tags. They simply tell what land the numbers are in. Numbers without subscripts are assumed to be in Ten Land.

The sequencing for both operations is the same: importing or exporting with Stage 1 or Stage 2 recording. Stage 1 recording means just write the answer. Stage 2 recording means "show your work": record each trade that was made to get the answer.
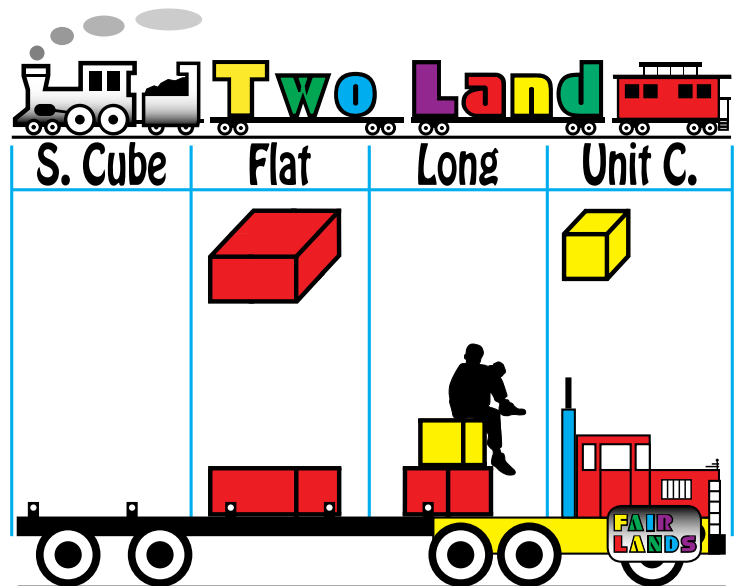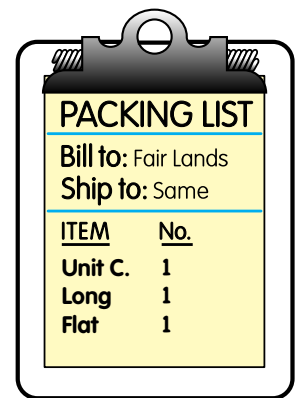
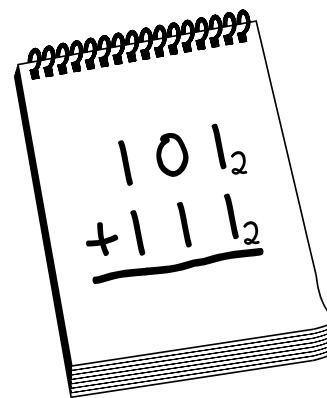## Importing with the Fair Lands Blocks

Materials

The same as for Fair Trades UP Penny Game in Two Land and Three Land.
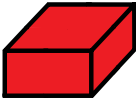
Lesson

Import (add) in Two Land and Three Land with the Fair Lands blocks. Do so in Ten Land and at least one other land with counters. If Ten Land blocks are available, do so with them before doing so with counters. Once students can "think blocks" and add in Two Land and Three Land, they can add in any land. All that changes in adding in different lands or bases is the number of things that make the next bigger thing.

Two Land Example

The top addend represents the blocks already in Fair Lands. The bottom one represents those on the truck and on the packing list that are being imported to Fair Lands.

$$\begin{array}{r} 1\ 0\ 1_2 \\ +\ 1\ 1\ 1_2 \\ \hline \end{array}$$

PACKING LIST

Bill to: Fair Lands
Ship to: Same

ITEM	No.
Unit C.	1
Long	1
Flat	1

S. Cube	Flat	Long	Unit C.

Two Land

S. Cube	Flat	Long	Unit C.
	🟥		🟨

Unloaded in Fair Lands

S. Cube	Flat	Long	Unit C.
	🟥	🟥	🟨

Too many Unit Cubes! So keeping it "simple," two Unit Cubes are traded for a Long.
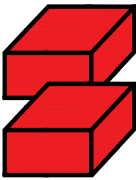
Two Land
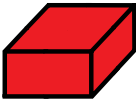
S. Cube	Flat	Long	Unit C.
	🟥	🟥	
	🟥	🟥	
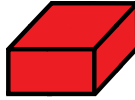
Too many Longs! So keeping it "simple," two Longs are traded for a Flat.

Two Land

S. Cube	Flat	Long	Unit C.
	🟥🟥		
	🟥		

Two Land

S. Cube	Flat	Long	Unit C.
🟥	🟥		

Safe at last! Stage 1 recording and Stage 2 recording for the event are shown below.

Ans. Only   *Ans. and Work*
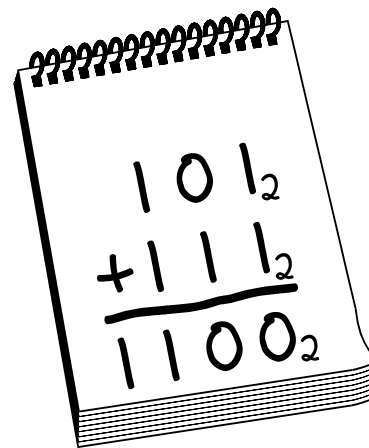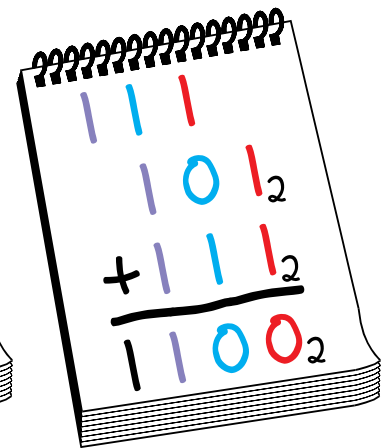
$$101_2 + 111_2 = 1100_2$$

Two Land Importing Practice

1. $110_2 + 11_2$ (ans. $1{,}001_2$)
2. $101_2 + 101_2$ (ans. $1{,}010_2$)
3. $1110_2 + 1011_2$ (ans. $11{,}001_2$)
4. $11110_2 + 1100_2$ (ans. $101{,}010_2$)
5. $1011_2 + 11010_2$ (ans. $100{,}101_2$)
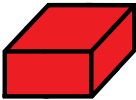6. $111111_2 + 1010_2$ (ans. $1{,}001{,}001_2$)

Three Land Example

The top addend represents the blocks already in Fair Lands. The bottom one represents those on the truck and on the packing list that are being imported to Fair Lands.

$$1012_3$$
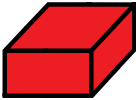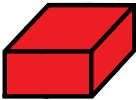$$+\ 221_3$$

PACKING LIST

Bill to: Fair Lands
Ship to: Same

ITEM	No.
Unit C.	1
Long	2
Flat	2

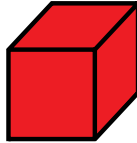| S. Cube	Flat	Long	Unit C.

Too many Longs! So keeping it "simple," three Longs are traded for a Flat.

S. Cube	Flat	Long	Unit C.

S. Cube	Flat	Long	Unit C.

Too many Flats! So three Flats are traded for a Super Cube.

S. Cube	Flat	Long	Unit C.

Unloaded in Fair Lands

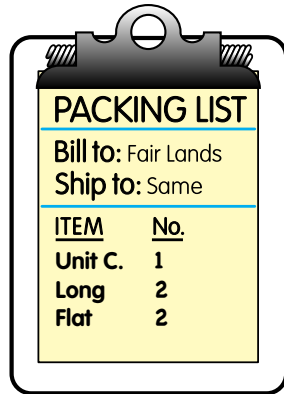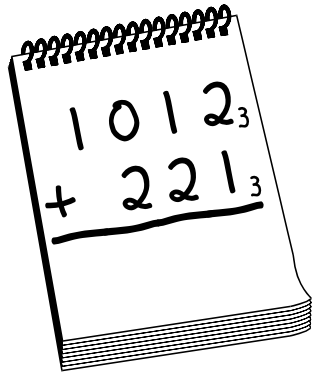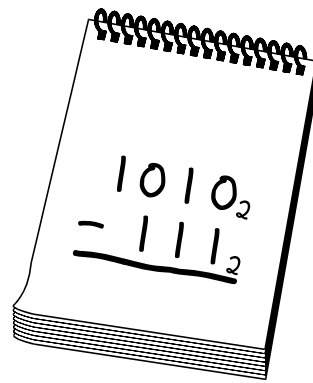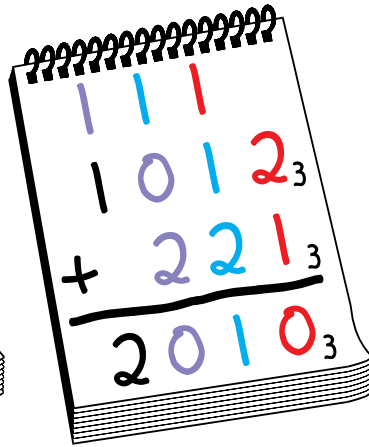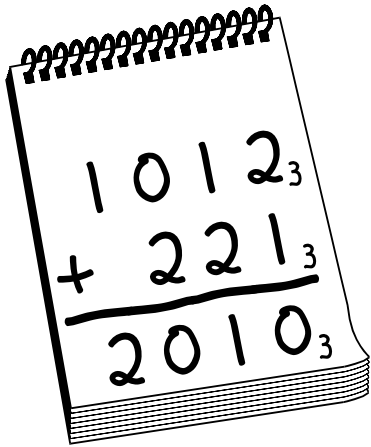| S. Cube	Flat	Long	Unit C.

Too many Unit Cubes! So keeping it "simple," three Unit Cubes are traded for a Long.

Safe at last! Stage 1 Recording and Stage 2 recording for the event are shown below.

12

$$1012_3 + 221_3 = 2010_3$$

$$1012_3 + 221_3 = 2010_3$$

$$1010_2 - 111_2$$

PURCHASE ORDER

Vendor: Fair Lands

Ship to: MIM

ITEM	No.
Unit C.	1
Long	1
Flat	1
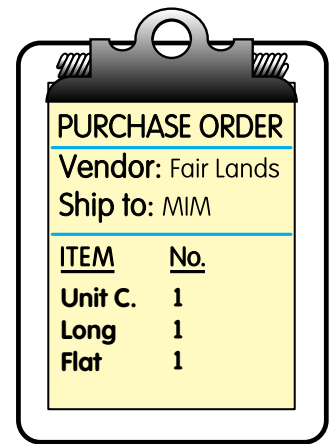
Three Land Importing Practice

1. $1211_3 + 121_3$ (ans. $2{,}102_3$)

2. $1221_3 + 122_3$ (ans. $2{,}120_3$)

3. $2102_3 + 12001_3$ (ans. $21{,}110_3$)

4. $122122_3 + 201_3$ (ans. $200{,}100_3$)

5. $22102_3 + 2202_3$ (ans. $102{,}011_3$)

6. $1222220_3 + 22_3$ (ans. $2{,}000{,}012_3$)

Exporting with the Fair Lands Blocks

Materials

The same as for Fair Trades DOWN Penny Game in Two Land and Three Land.

Lesson

Export (subtract) in Two Land and Three Land with the Fair Lands blocks. Do so in Ten Land and at least one other land with counters. If Ten Land blocks are available, do so with them before doing so with counters.

Once elementary school students can "think blocks" and subtract in Two Land and Three Land, they can subtract in any land. All that changes in subtracting in different lands or bases is how many things make the next bigger thing.

Two Land Example

The minuend represents the blocks already in Fair Lands, the subtrahend those on the purchase order to be put on the truck and exported from Fair Lands.

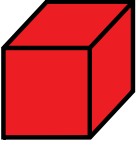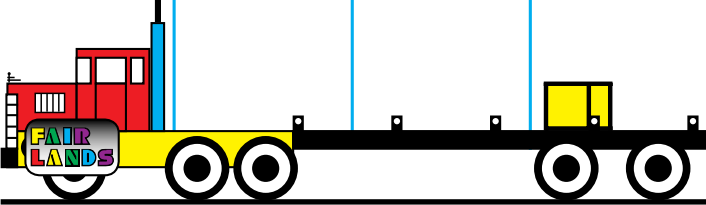| S. Cube	Flat	Long	Unit C.
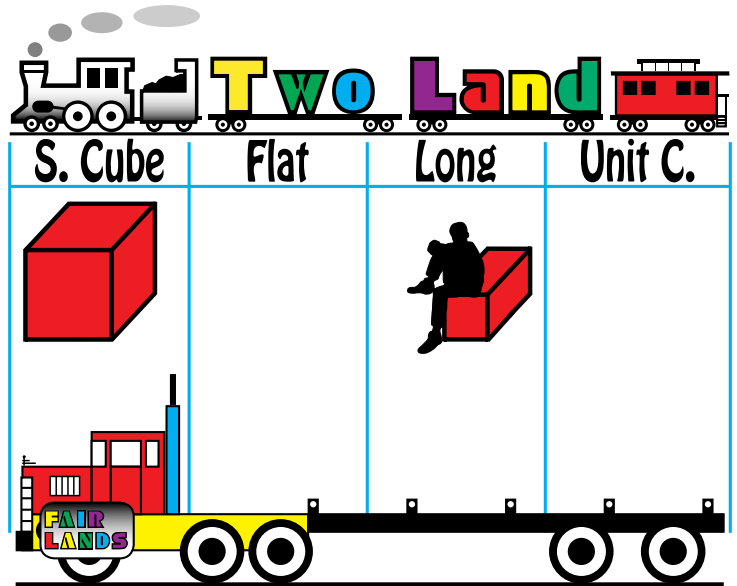
Must put a Unit Cube on the truck. No Unit Cubes, so the Long is traded for two Unit Cubes, one of which is put on the truck.
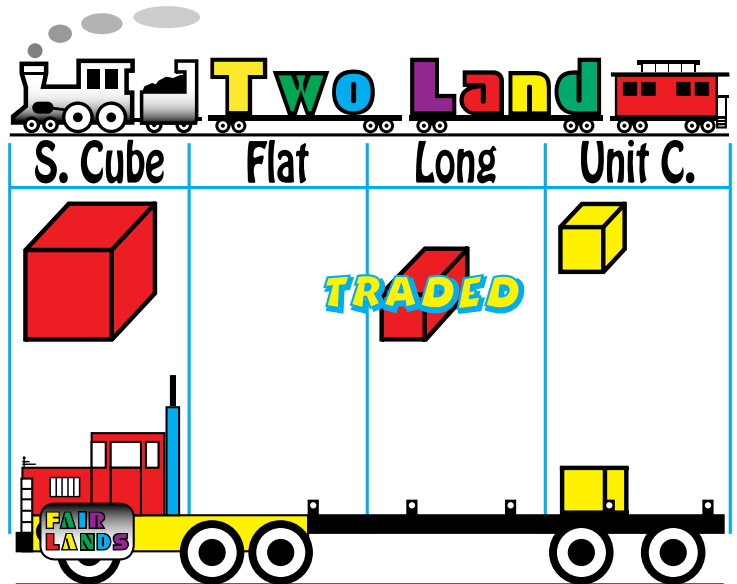
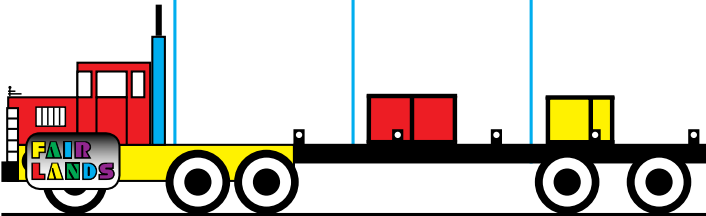| S. Cube	Flat	Long	Unit C.

One Super Cube and one Unit Cube are left in Fair Lands.

13

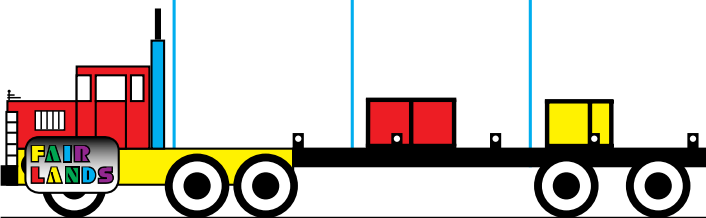| S. Cube	Flat	Long	Unit C.

Must put a Long on the truck. No Longs, so the Super Cube is traded for two Flats. Then one of the Flats is traded for two Longs, one of which is put on the truck.
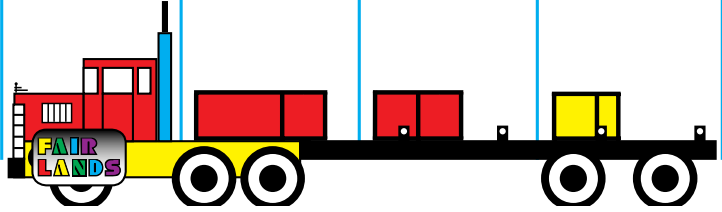
S. Cube	Flat	Long	Unit C.

One Flat, one Long, and one Unit Cube are left in Fair Lands.

S. Cube	Flat	Long	Unit C.

Must put the Flat on the truck. One Long and one Unit are left in Fair Lands.

S. Cube	Flat	Long	Unit C.

Order filled. Stage 1 Recording and Stage 2 recording for the problem are shown below.

Ans. Only   *Ans. and Work*
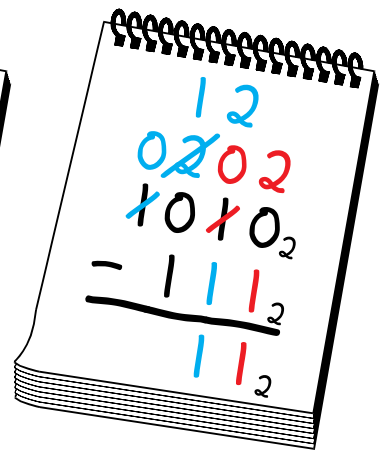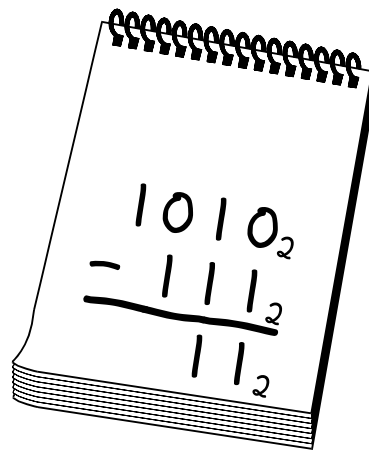
$$1010_2 - 111_2 = 11_2$$

Two Land Exporting Practice

1. $110_2 - 11_2$ (ans. 11_2)

2. $1001_2 - 101_2$ (ans. 100_2)

3. $11001_2 - 1011_2$ (ans. $1{,}110_2$)

4. $101010_2 - 11110_2$ (ans. $1{,}100_2$)

5. $100101_2 - 11010_2$ (ans. $1{,}011_2$)

6. $1001001_2 - 111111_2$ (ans. $1{,}010_2$)

Three Land Example

The minuend represents the blocks already in Fair Lands, the subtrahend those on the purchase order to be put on the truck and exported from Fair Lands.

$$2021_3$$
$$-1212_3$$

ITEM	No.
Unit C.	2
Long	1
Flat	2
Cube	1

Two Super Cubes, one Long, and two Unit Cubes are left in Fair Lands.

Must put two Unit Cubes on the truck. Only have one, so a Long is traded for three Unit Cubes, two of which are put on the truck.

Must put the Long on the truck. Two Super Cubes and two Unit Cubes are left in Fair Lands.

15

Must put two Flats on the truck. No Flats, so a Super Cube is traded for three Flats, two of which are put on the truck.

One Super Cube, one Flat, and two Unit Cubes are left in Fair Lands.

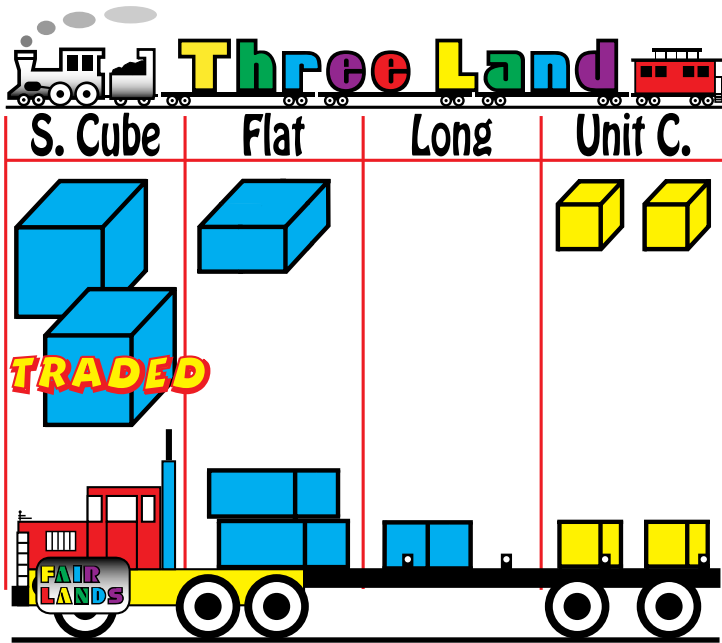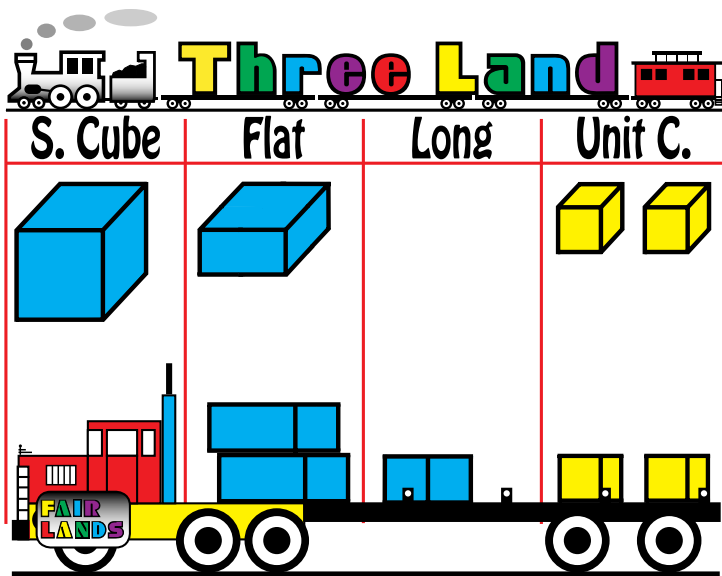Must put the Super Cube on the truck. One Flat and two Unit Cubes are left in Fair Lands.
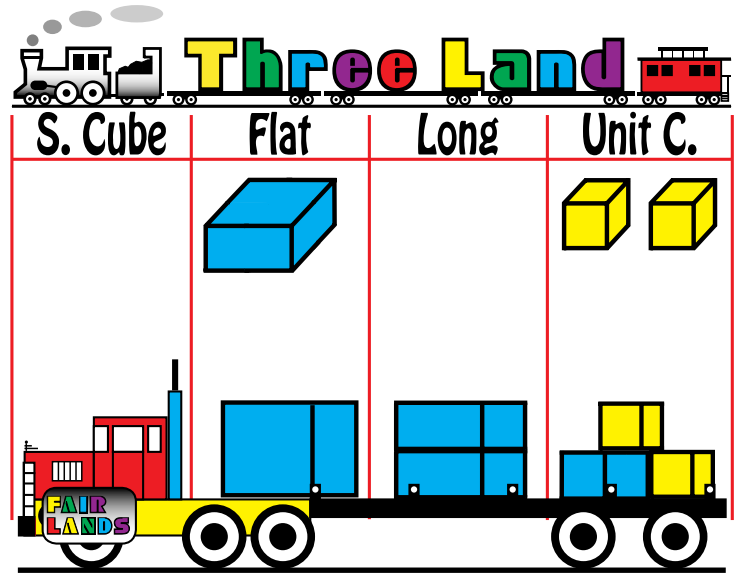
Order filled. Stage 1 Recording and Stage 2 recording for the event are shown below.

Ans. Only   *Ans. and Work*
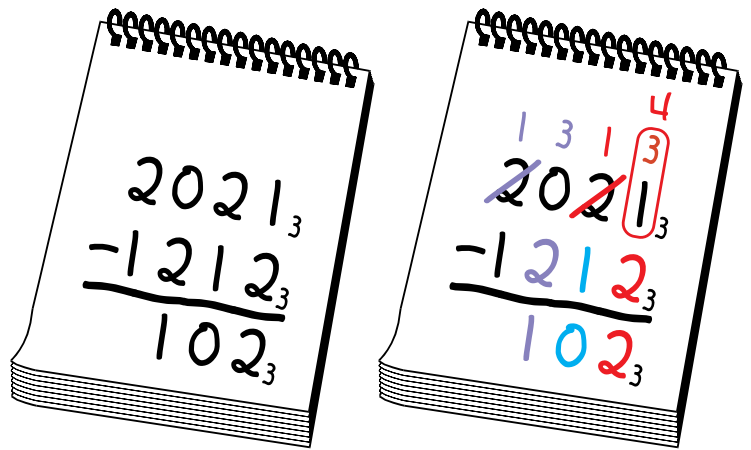
Three Land Exporting Practice

1. $1201_3 - 212_3$ (ans. 212_3)

2. $2012_3 - 122_3$ (ans. $1{,}120_3$)

3. $21110_3 - 12001_3$ (ans. $2{,}102_3$)

4. $200100_3 - 122122_3$ (ans. 201_3)

5. $102011_3 - 22102_3$ (ans. $2{,}202_3$)

6. $2000012_3 - 1222220_3$ (ans. 22_3)

Are the Fair Lands Blocks Just for Children?

Years ago, about the time of 'Rithmetic in Residence, the author of this booklet, Paul Shoecraft, taught a night class at Arizona State Prison, a federal penitentiary in Florence. The class covered arithmetic and elementary algebra and met once a week for about 2 hours for 14 weeks. About 20 men took the class, ranging in age from 20-50 and in incarceration time from 1-20 years. Instruction was individualized in a continual progress format.

While subtracting mixed numbers, Paul noticed some of the men just sprinkling 1s about when they had to "borrow":

$$5\frac{3}{8}$$
$$-2\frac{7}{8}$$

$$\longrightarrow$$

$$5\overset{4}{\cancel{5}}\frac{{}^{1}3}{8}$$
$$-2\frac{7}{8}$$
$$2\frac{6}{8} = 2\frac{3}{4}$$

He addressed the error with chalk-and-talk and explained that if working with, say, eighths, that they had to borrow eight eighths, not ten eighths as they were doing when they just stuck a one in front of a numerator like the 3 in the example, but old habits die hard, and they didn't get it. What they got was another rote, meaningless procedure to replace the rote, meaningless one they'd been using, and pretty soon they were back to doing what they'd been doing. They didn't grasp that fractions were in "lands" determined by their denominators and that the lands governed the trades that could be made.

Paul was beside himself. He reckoned the men needed to work with the Fair Lands blocks but dared not offend them by thrusting upon them what looked like children's building blocks. Although none of the men had ever confronted

him in the slightest, a prison is a dangerous place, and when tempers flare, men die, so he explained his angst to them and sought their permission to teach them with the blocks.

"Gentlemen, I see the problem you're having with fractions. I gave you my best lecture when I talked about what you were doing wrong, but it didn't connect with you. The only way I know to make it connect is with some colored blocks (Cuisenaire multi-base arithmetic blocks at the time) I use when I teach fractions to children. I would like to use the blocks with you, but I don't want to offend you. What do you want me to do?"

The men discussed his wish and arrived at a consensus:

"Bring the blocks. Just close the door to the classroom."

Paul brought the blocks for two sessions. The result was *SUCCESS!*

$$5\frac{3}{8}$$
$$-2\frac{7}{8}$$

$$\longrightarrow$$

$$5\overset{4}{\cancel{5}}\frac{\overset{11}{\boxed{\frac{8}{3}}}}{8}$$
$$-2\frac{7}{8}$$
$$2\frac{4}{8} = 2\frac{1}{2}$$

He was surprised, though, when afterward the men said thank you. *Thank you!* These tough men in a tough place THANKED him for a math lesson for kids.

Terminology

The following is a list of the vocabulary pertaining to Fair Lands that was used in this booklet. It's all right to invent your own in working with the Fair Lands blocks. The blocks and the way they're made does most of the teaching, not the talk.

Exporting

Modeling subtraction as a business transaction— a sale—in Fair Lands. The blocks or counters already on a Fair Lands activity board show what's in inventory before the sale. Those on the invoice show what's to be taken out of inventory to fulfill the sale.

Fair Land

A trading center for trades based on numbers and established exchange rates.

Fair Trade

What the Law of the Land dictates. In Two Land, the fair trades would be two Unit Cubes for a Long, two Longs for a Flat, two Flats for a Super Cube, and so on up to two Super Flats for a Mega Cube. In Three Land they would be three Unit Cubes for a Long, three Longs for a Flat, three Flats for a Super Cube, and so on up to three Super Flats for a Mega Cube. In Ten Land the fair trades up to a thousand would be ten ones for a ten, ten tens for a hundred, and ten hundreds for a thousand.

Importing

Modeling addition as a business transaction— a purchase—in Fair Lands. The blocks or counters already on a Fair Lands activity board show what's in inventory. Those on the purchase order show what's to be added to the inventory.

KISS

An acronym for "Keep it safe and simple," meaning keep your activity board safe and trades simple.

Land

An indicator that the numbers governing a trade are sanctioned—that when a trade can be made, it must be made, not may be made.

Law of the Land

Used only with Two Land, Three Land, and other place value numeration systems like Ten Land where it's required that trades be made, not lands like those for money, time, measurement, and fractions where trades are optional.

The Law of the Land in Two Land is "Never have two or more alike." That in Three Land "Never have three or more alike." That in Ten Land "Never have ten or more alike."

Safe

Not in violation of the Law of the Land. Being "safe" in Fair Lands acknowledges the number that's raised to consecutive powers to designate place value in a land. In Two Land or base 2, the number is two. In Three Land or base 3, it's three. In Ten Land or base 10, it's ten. In each case, the Law of the Land is that the base number cannot be equaled or exceeded by that many blocks or counters all the same, that is, in the same place (column) on a Fair Lands activity board.

Simple (as in "keep trades simple")

In importing and exporting, keeping trades "simple" means always trading the smallest and fewest blocks or counters possible. The "smallest" requirement forces working from right to left on a Fair Lands activity board. The "fewest" one ensures working column by column and not skipping over columns.

Stage 1 Recording

Recording only the answer to an importing or exporting problem. This requires that students can represent a Two Land or Three Land numeral with blocks or counters. Once they can do this, they can be directed to centers where they can work from written instructions like 1012+112 or 10012–1102 for the Two Land blocks or 11103+2023 or 2013–123 for the Three Land blocks.

Stage 2 Recording

Recording each trade in an importing or exporting problem. Stage 2 recording models

the algorithms—the step-by-step procedures by which we add or subtract. It means "Show your work."

Trading "Down"

Trading down is the reverse of trading up. It's exchanging big things for little things, like one Long for two Unit Cubes in Two Land, one Flat for three Longs in Three Land, or one hunded for ten tens in Ten Land (or a quarter for five nickels, a half hour for 30 minutes, one yard for three feet, or two thirds for four sixths).

Trading "Place"

A place designated by the Trading Place Card where the Fair Lands blocks not on an activity board are kept.

Trading "Up"

Trading up is exchanging little things for big things, like two Unit Cubes for a Long in Two Land, three Longs for a Flat in Three Land, or ten hundreds for a thousand in Ten Land (or two nickels for a dime, 365 days for a year, 100 cm for 1 m, or five tenths for one half).

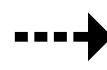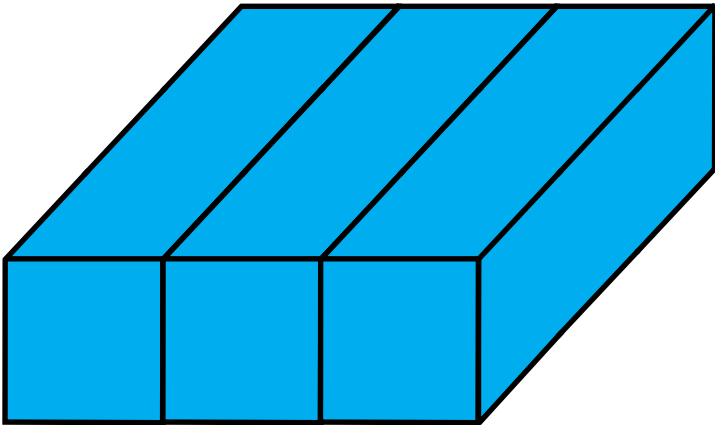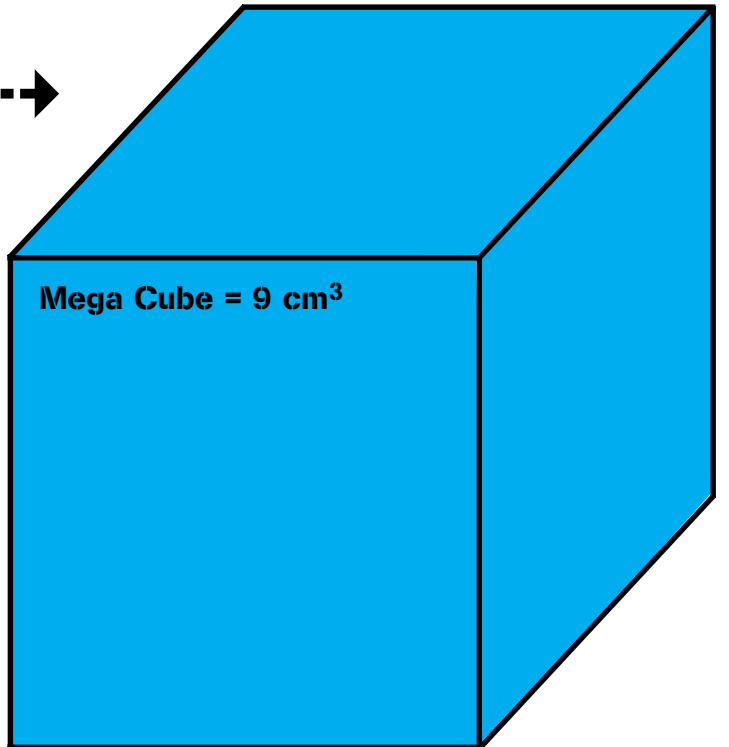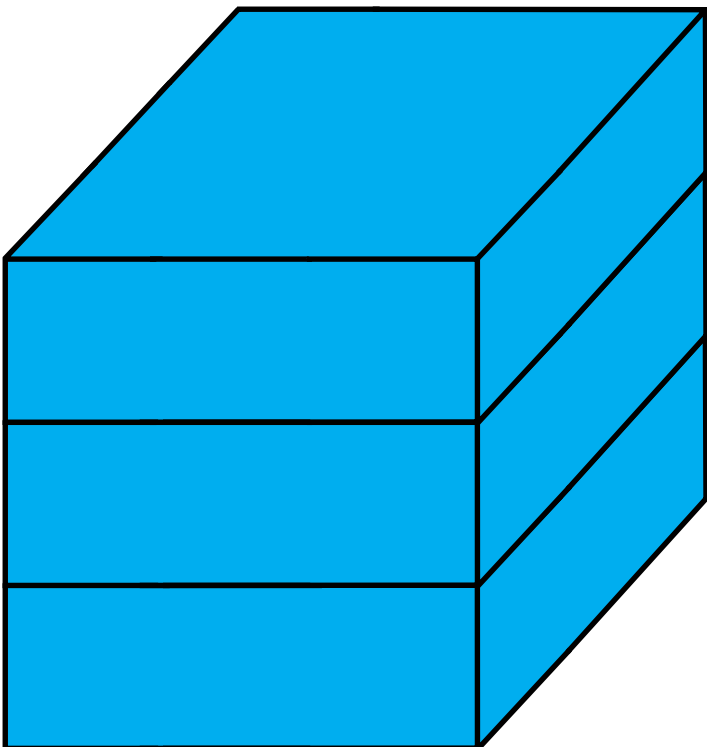**TWO LAND:** Two the same make the next bigger block.
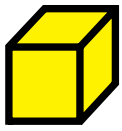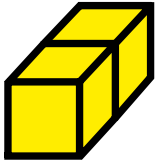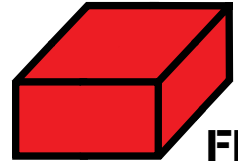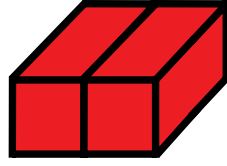
Unit Cube = 1 cm^3
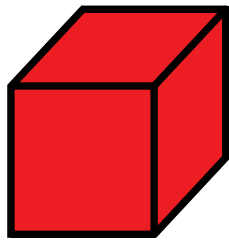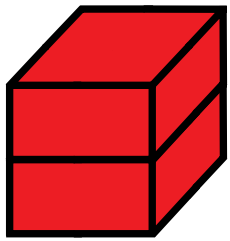
Long

Flat

Super Cube = 2 cm^3
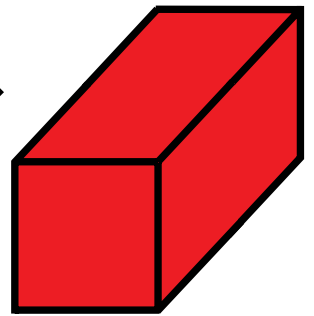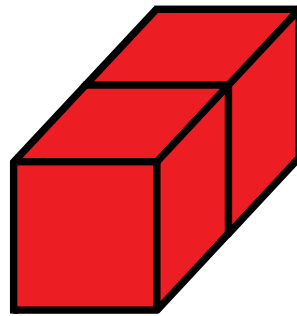
Super Long
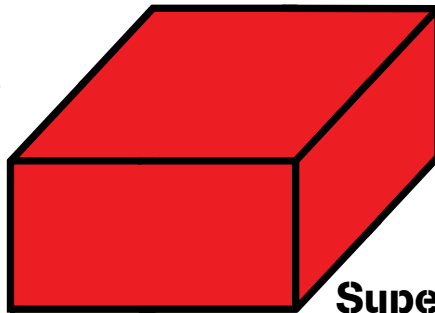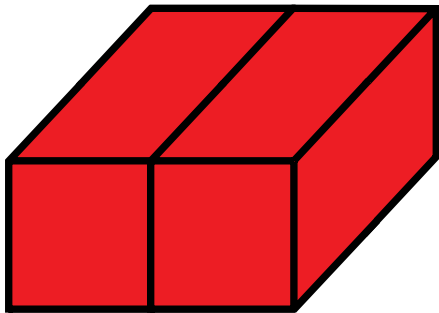
Super Flat
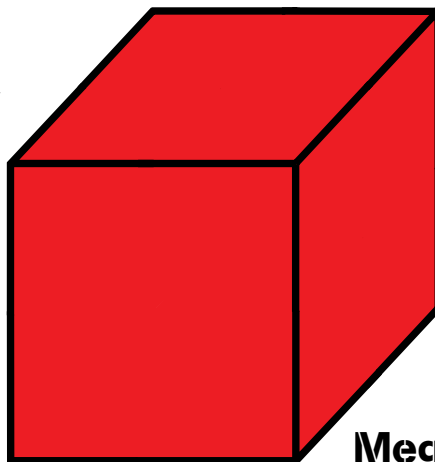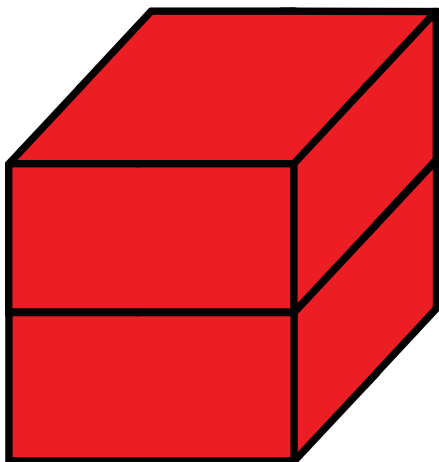
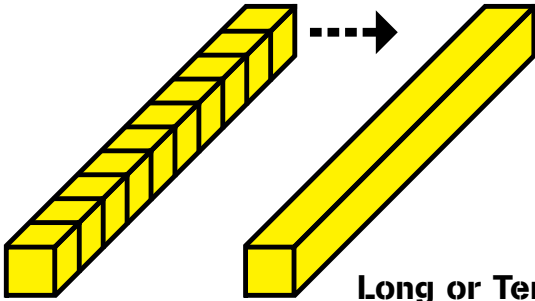Mega Cube = 4 cm^3

THREE LAND: Three the same make the next bigger block.
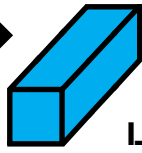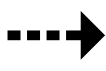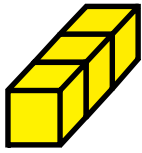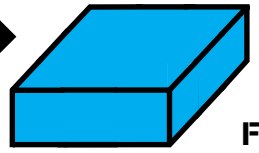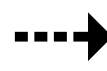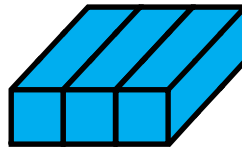
Unit Cube = 1 cm³
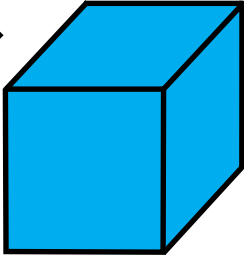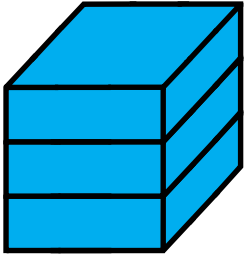
Long

Flat

Super Cube = 3 cm³
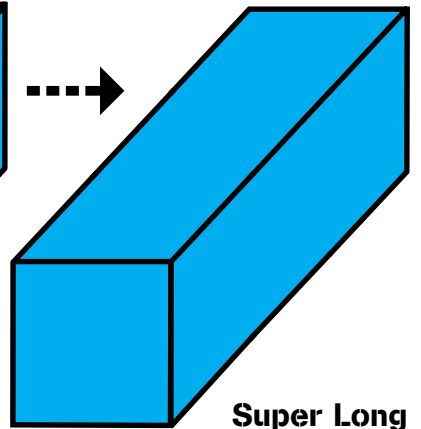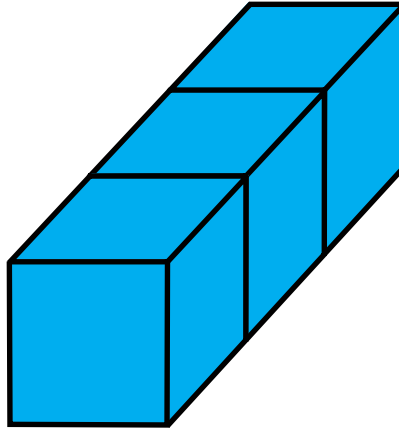
Super Long
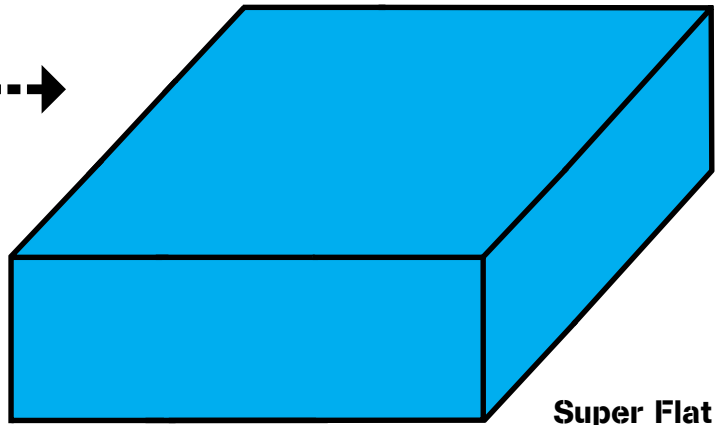
Super Flat
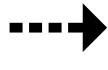
Mega Cube = 9 cm³

TEN LAND: Ten the same make the next bigger block.

Unit Cube or One = 1 cm³

Long or Ten Rod = 10 cm x 1 cm x 1 cm
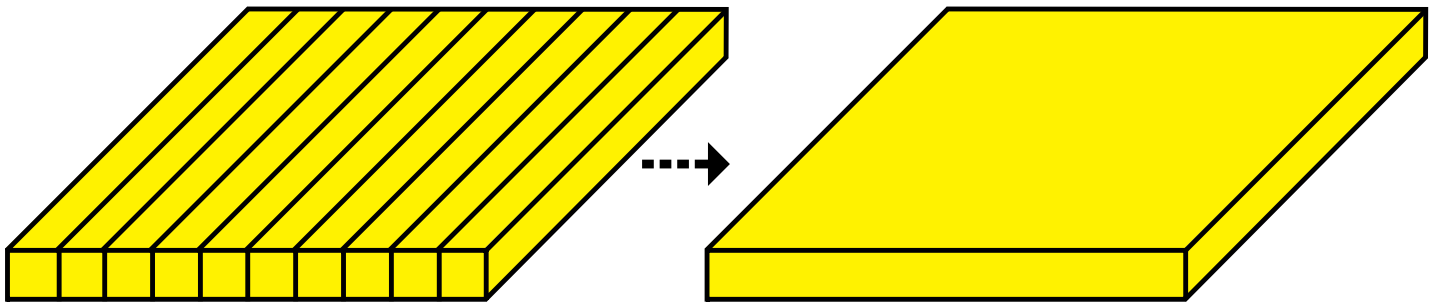
Flat or Hundred Square = 10 cm x 10 cm x 1 cm
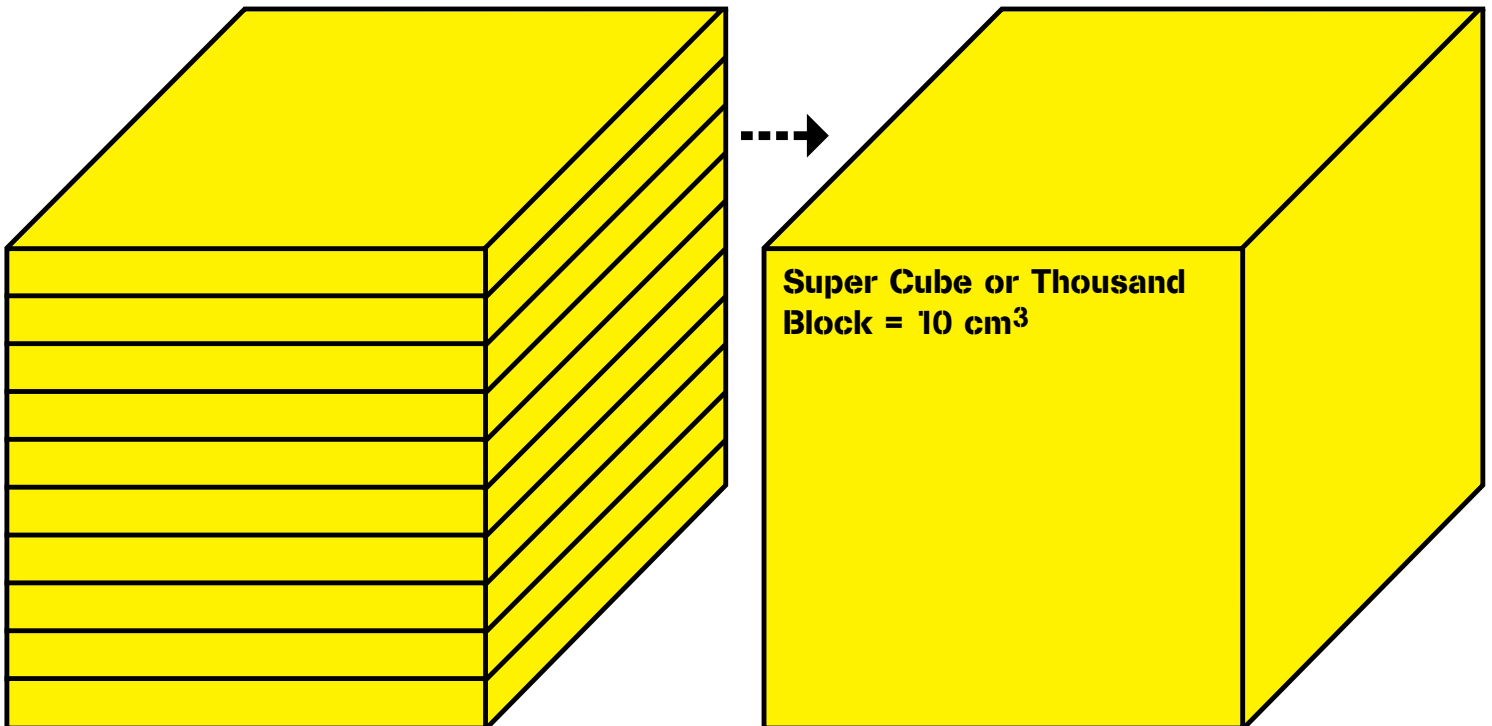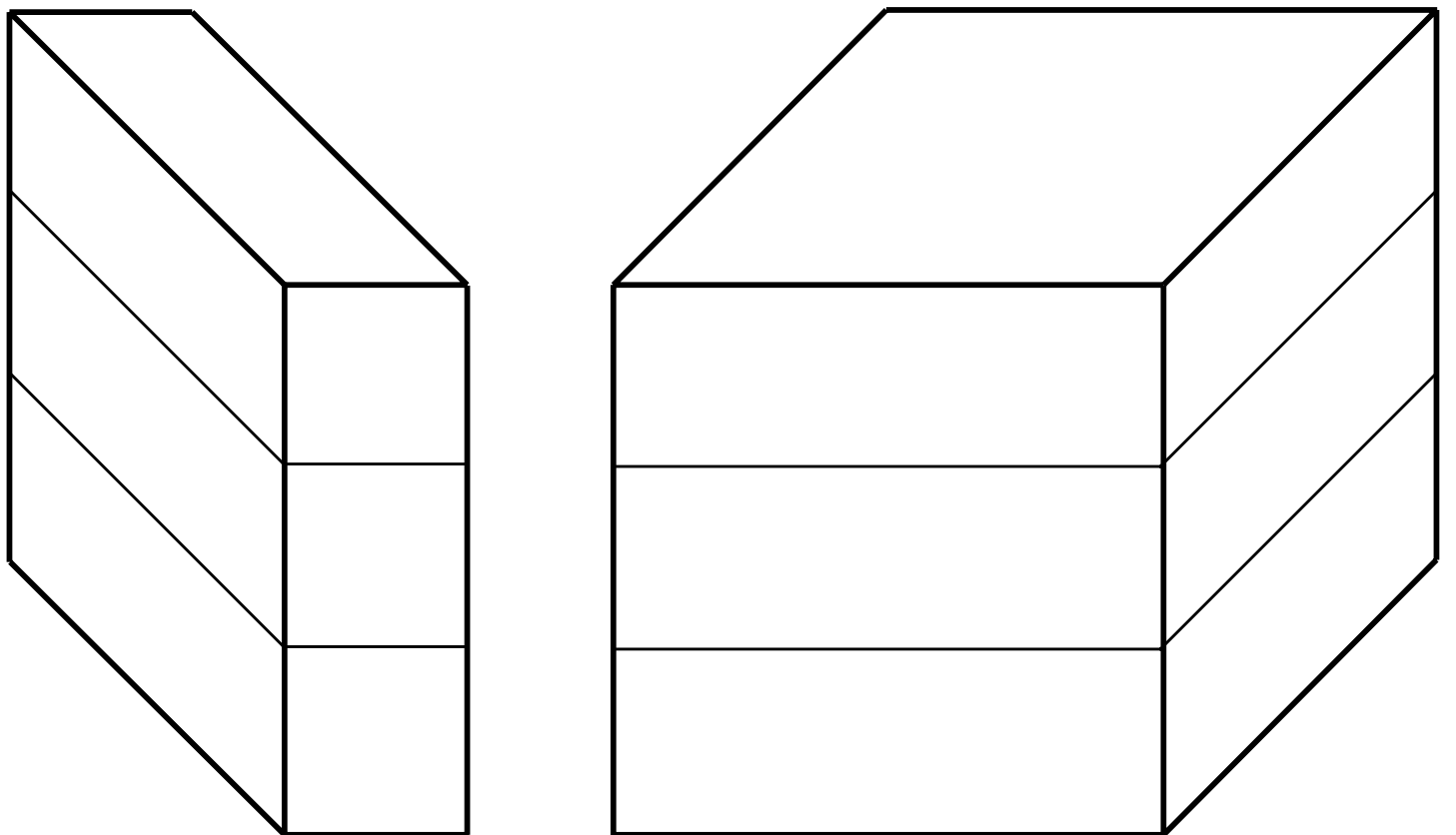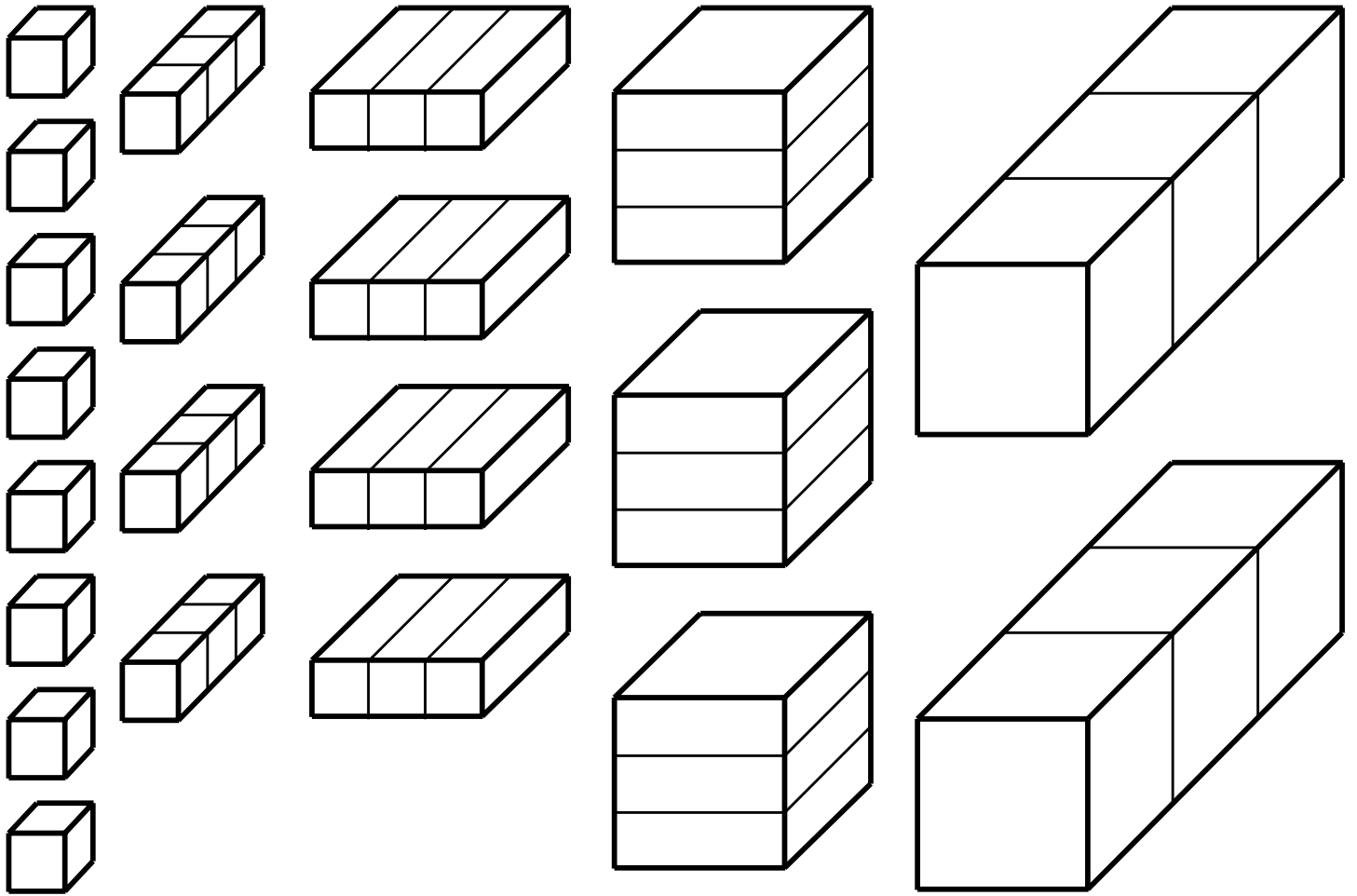
Super Cube or Thousand Block = 10 cm³

Template for Making Paper Models of the Two Land Blocks
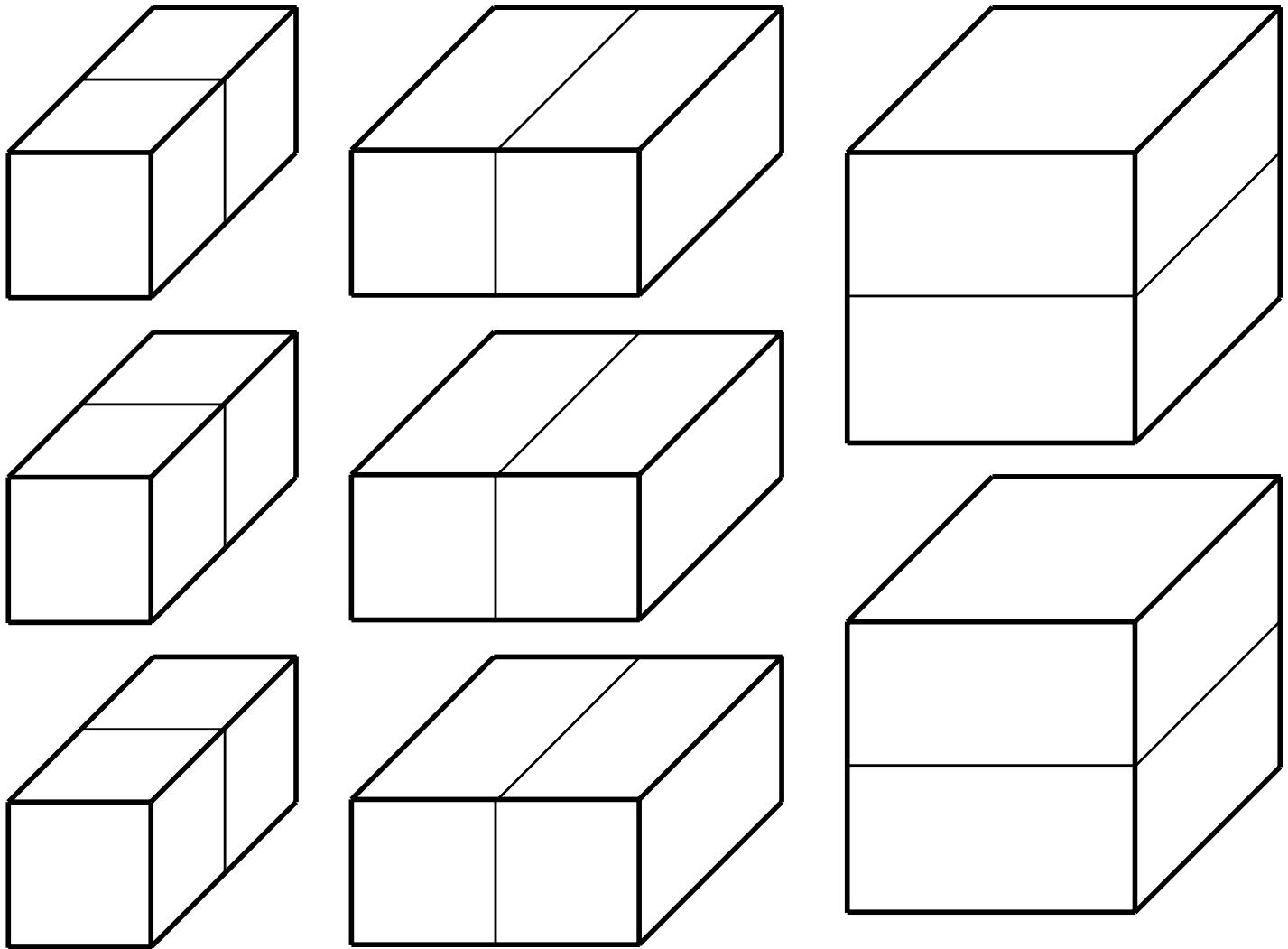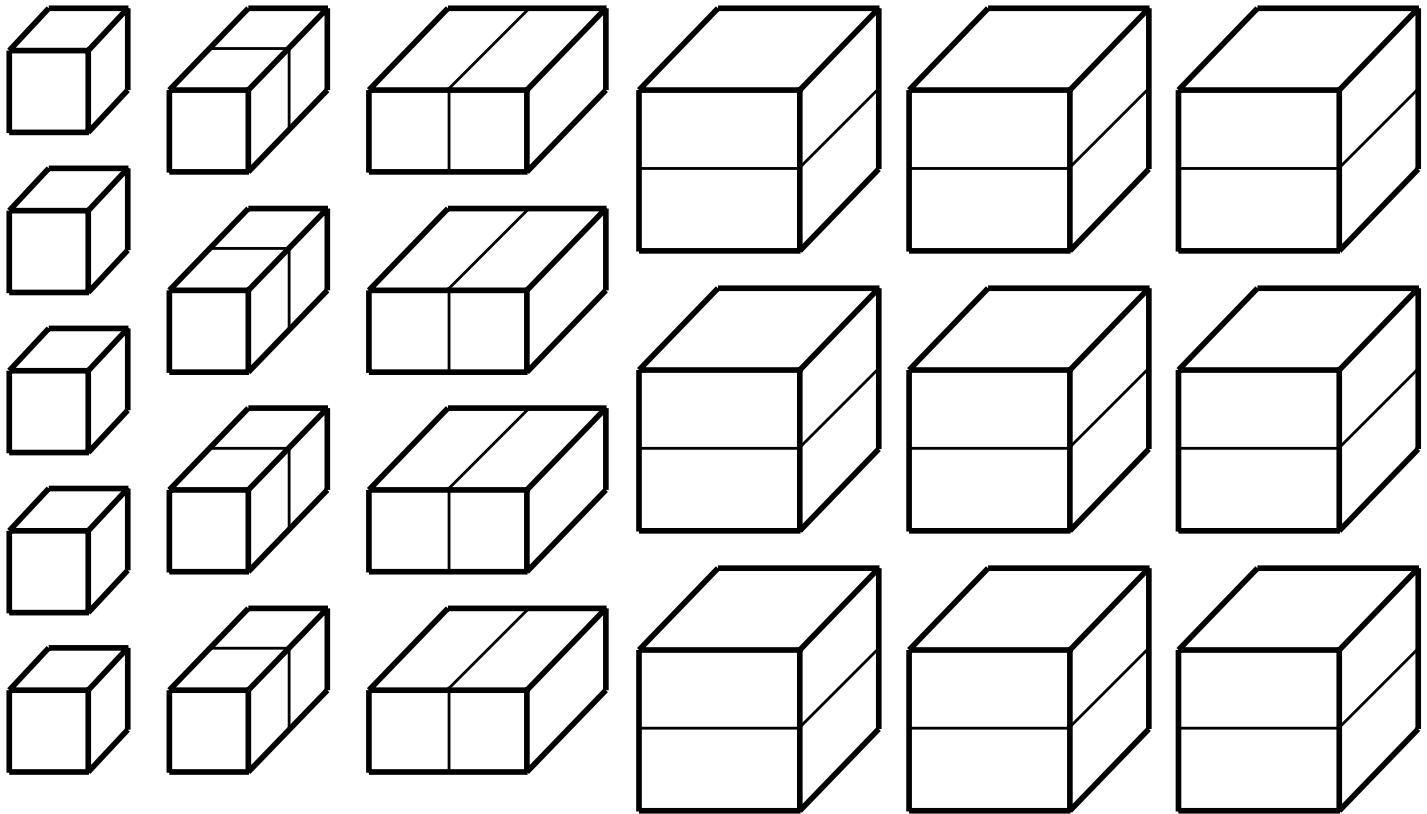
Template for Making Paper Models of the Three Land Blocks

Template for Making Paper Models of the Ten Land Blocks