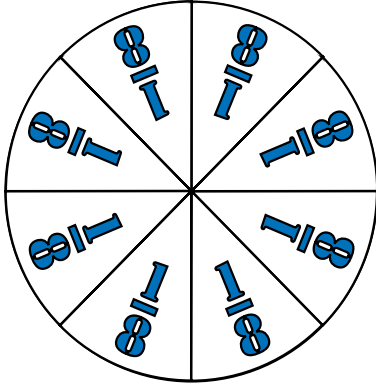
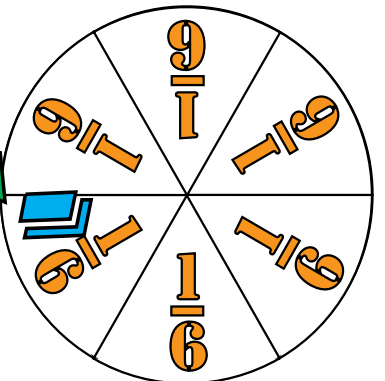


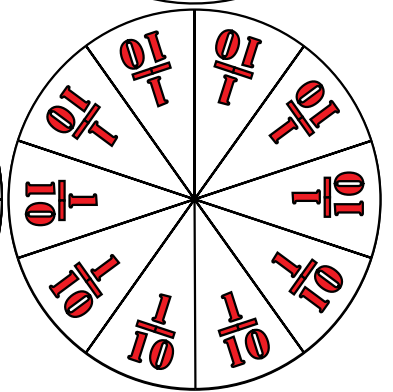
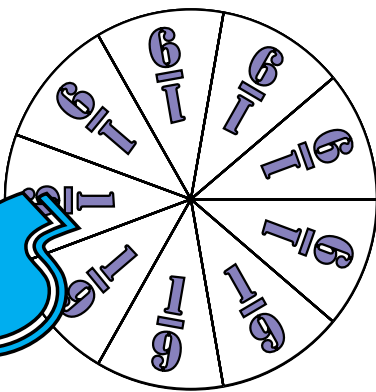
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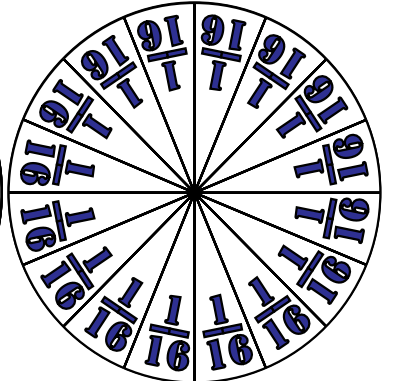
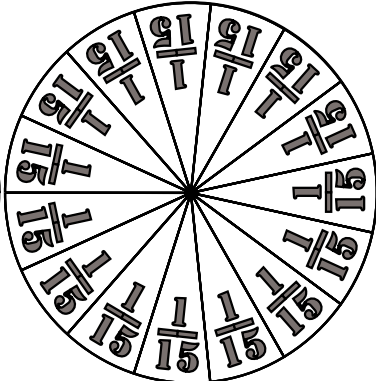
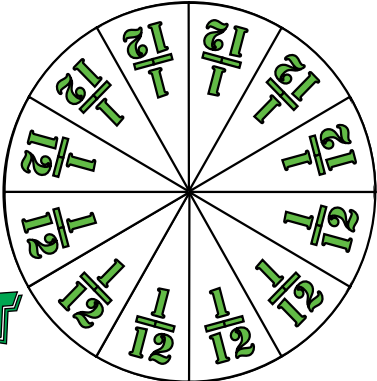
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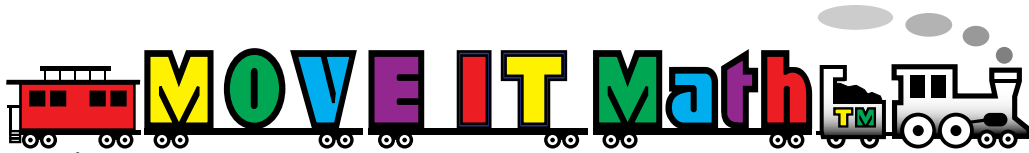


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# REAL FRACTIONS

## Using Fraction Circles to Discover the Rules for Working with Fractions

Kindergarten - Grade 8

MOVE IT Math™

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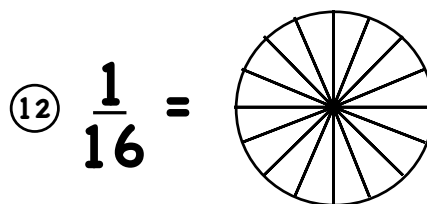
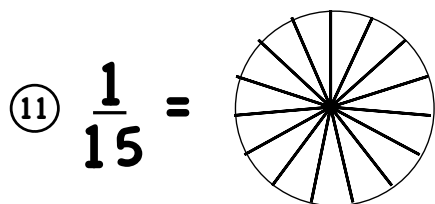
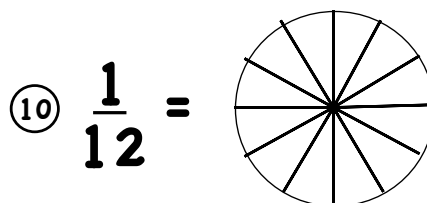
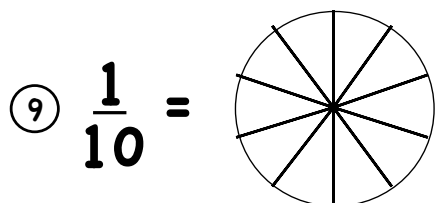
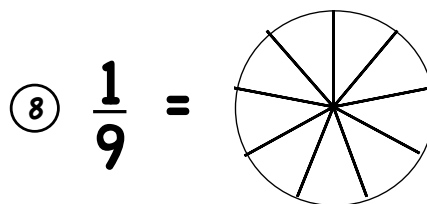
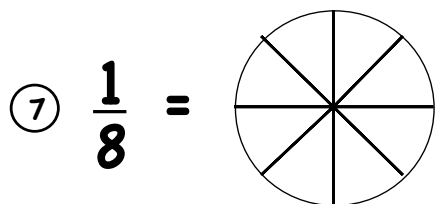
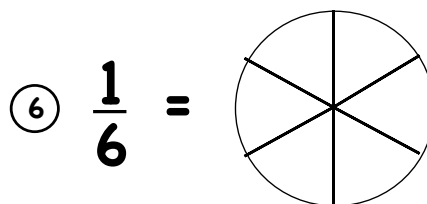
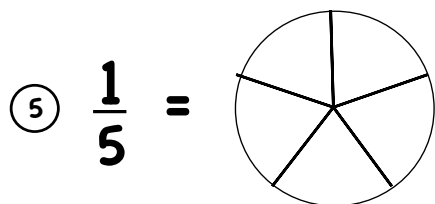
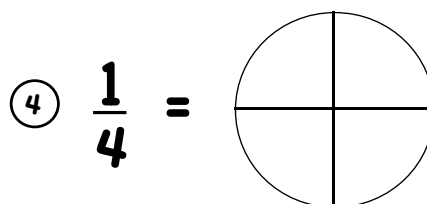
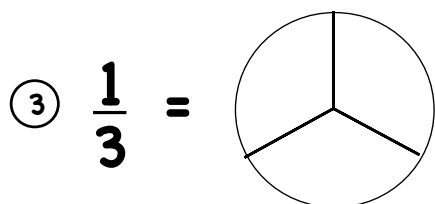
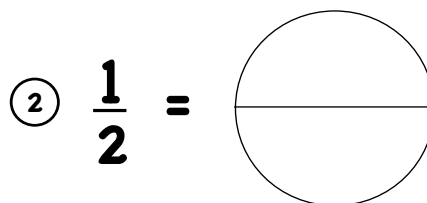
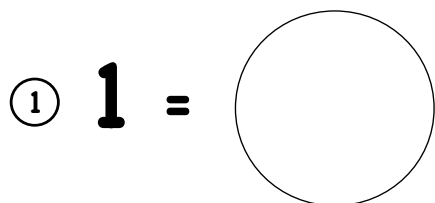
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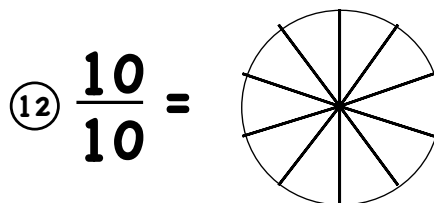
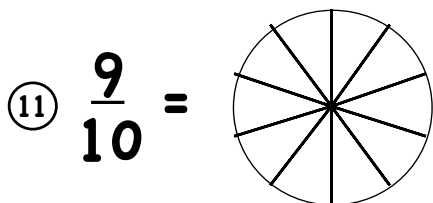
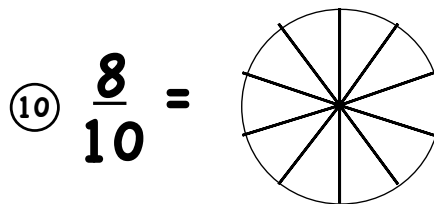
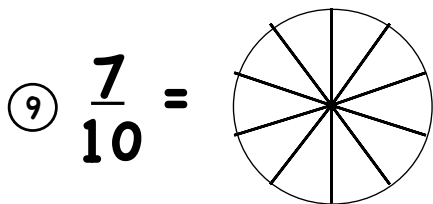
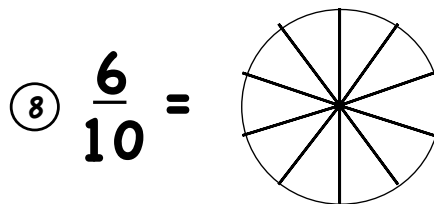
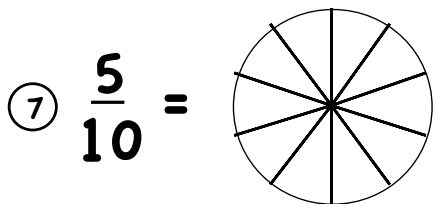
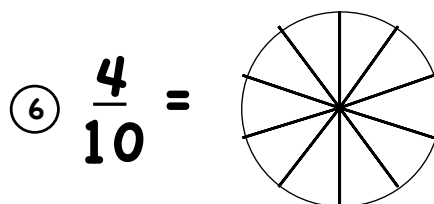
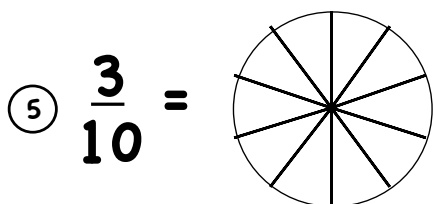
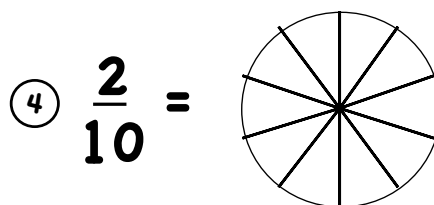
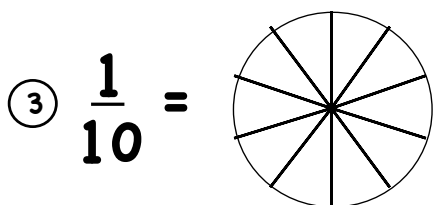
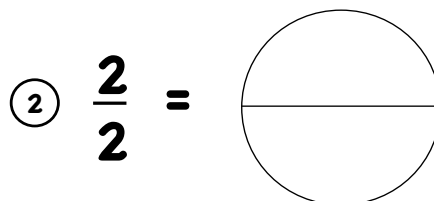
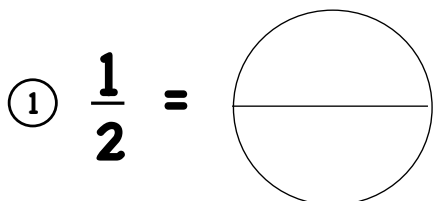
## Exploring Fractions with Fraction Circles: Constant Numerators

Show each fraction with Fraction Circles. Color the fraction picture to match.

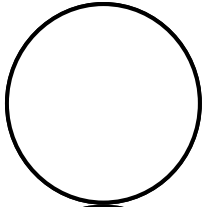


## Exploring Fractions with Fraction Circles: Constant Denominators

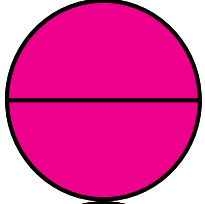
Show each fraction with Fraction Circles. Color the fraction picture to match.



## Comparing Fractions with Fraction Circles



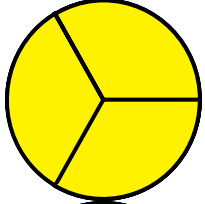
Use Fraction Circles to decide less than (<), greater than (>), or equal (=).



①  $\frac{1}{3} \bigcirc \frac{2}{3}$

②  $\frac{4}{9} \bigcirc \frac{7}{9}$

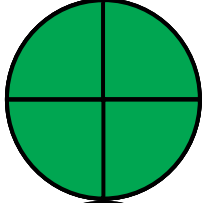
③  $\frac{9}{16} \bigcirc \frac{15}{16}$



④  $\frac{1}{2} \bigcirc \frac{1}{3}$

⑤  $\frac{3}{4} \bigcirc \frac{3}{5}$

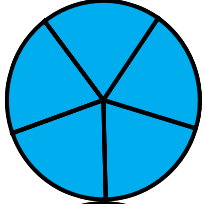
⑥  $\frac{11}{15} \bigcirc \frac{11}{16}$



⑦  $\frac{1}{2} \bigcirc \frac{4}{8}$

⑧  $\frac{1}{5} \bigcirc \frac{2}{10}$

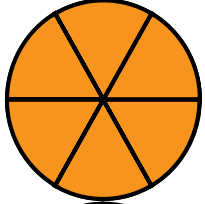
⑨  $\frac{3}{4} \bigcirc \frac{9}{12}$



⑬  $\frac{1}{2} \bigcirc \frac{5}{8}$

⑭  $\frac{1}{5} \bigcirc \frac{2}{9}$

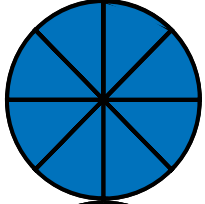
⑮  $\frac{1}{8} \bigcirc \frac{2}{15}$



⑯  $\frac{2}{3} \bigcirc \frac{3}{4}$

⑰  $\frac{4}{5} \bigcirc \frac{5}{6}$

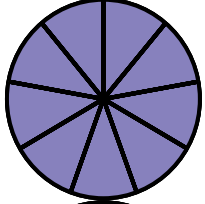
⑱  $\frac{7}{8} \bigcirc \frac{14}{15}$



⑲  $\frac{2}{5} \bigcirc \frac{3}{10}$

⑳  $\frac{2}{3} \bigcirc \frac{7}{12}$

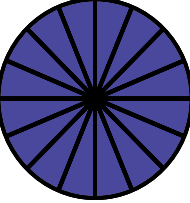
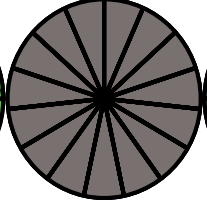
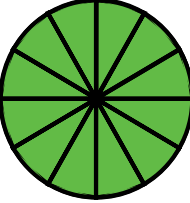
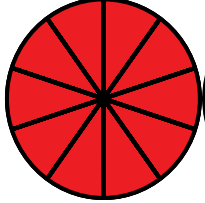
㉑  $\frac{3}{4} \bigcirc \frac{11}{16}$



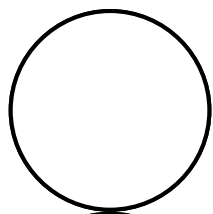
㉒  $\frac{2}{3} \bigcirc \frac{5}{6}$

㉓  $\frac{3}{4} \bigcirc \frac{5}{8}$

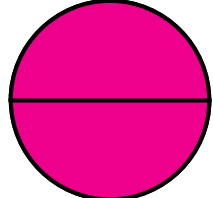
㉔  $\frac{3}{5} \bigcirc \frac{9}{15}$



## Ordering Fractions with Fraction Circles

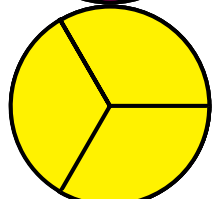


Use Fraction Circles to order each set of fractions from smallest to biggest.



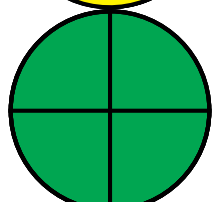
$$\textcircled{1} \quad \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$$

$$\textcircled{2} \quad \frac{1}{2}, \frac{1}{4}, \frac{7}{12}$$



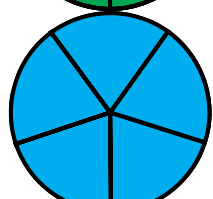
$$\textcircled{3} \quad \frac{2}{9}, \frac{2}{5}, \frac{2}{3}$$

$$\textcircled{4} \quad \frac{1}{2}, \frac{5}{8}, \frac{7}{16}$$



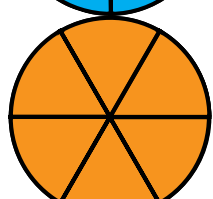
$$\textcircled{5} \quad \frac{3}{4}, \frac{3}{5}, \frac{3}{8}$$

$$\textcircled{6} \quad \frac{1}{3}, \frac{5}{12}, \frac{4}{15}$$



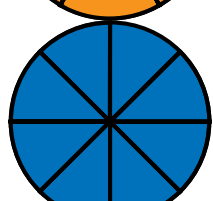
$$\textcircled{7} \quad \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$$

$$\textcircled{8} \quad \frac{2}{3}, \frac{5}{9}, \frac{11}{15}$$



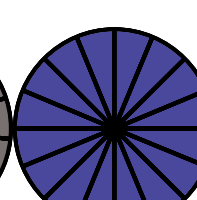
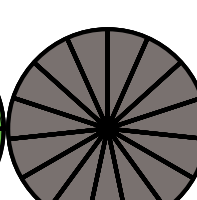
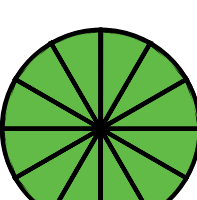
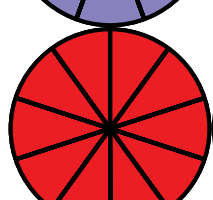
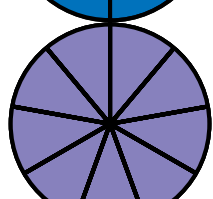
$$\textcircled{9} \quad \frac{5}{8}, \frac{3}{8}, \frac{1}{8}$$

$$\textcircled{10} \quad \frac{3}{4}, \frac{7}{8}, \frac{13}{16}$$



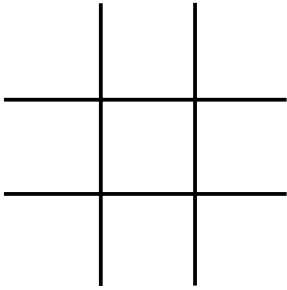
$$\textcircled{11} \quad \frac{2}{9}, \frac{5}{9}, \frac{8}{9}$$

$$\textcircled{12} \quad \frac{4}{5}, \frac{7}{10}, \frac{11}{15}$$

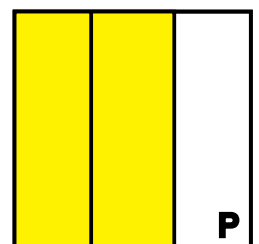
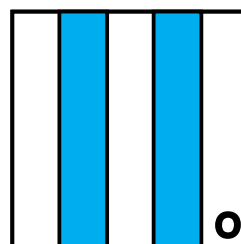
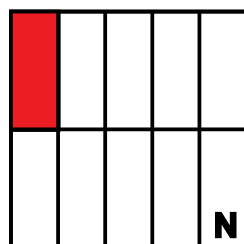
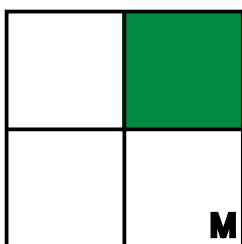
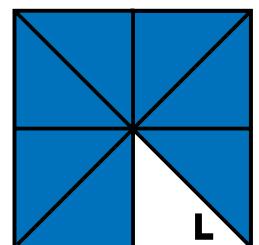
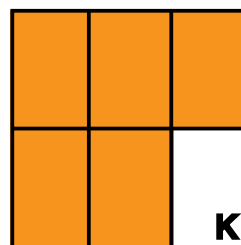
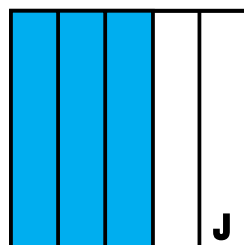
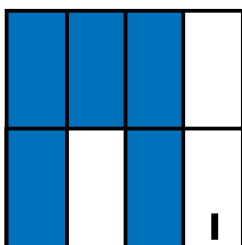
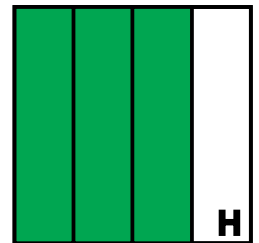
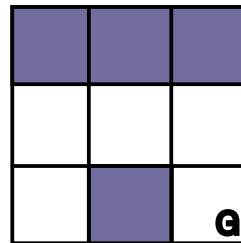
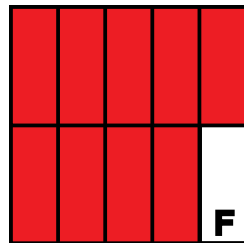
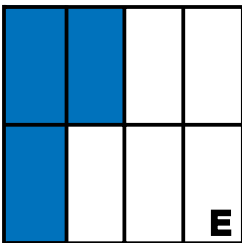
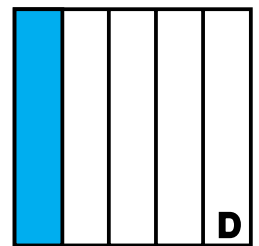
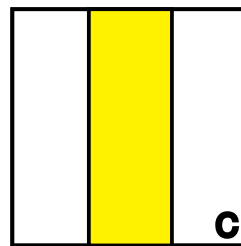
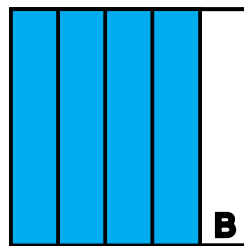
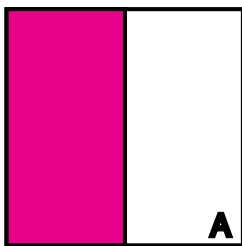


## Fraction Tic-Tac-Toe

Fill in a tic-tac-toe grid with any nine of the fractions below. As the squares below are called by letter, cross out the fractions in the grid that correspond to how much of the squares are shaded. For example, if C is called, cross out  $\frac{1}{3}$  (for 1 of 3 regions shaded). If E is called, cross out  $\frac{3}{8}$  (for 3 of 8 regions shaded). The first to cross out three fractions in a row horizontally, vertically, or diagonally wins.



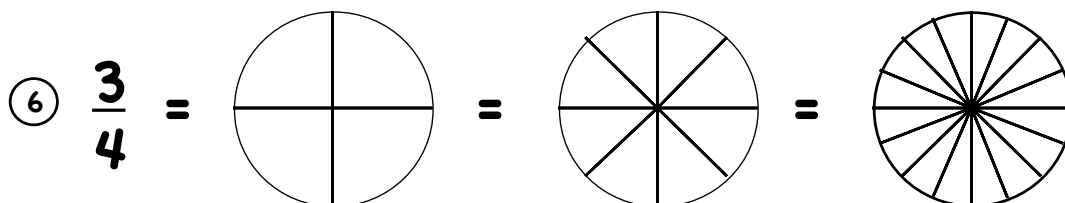
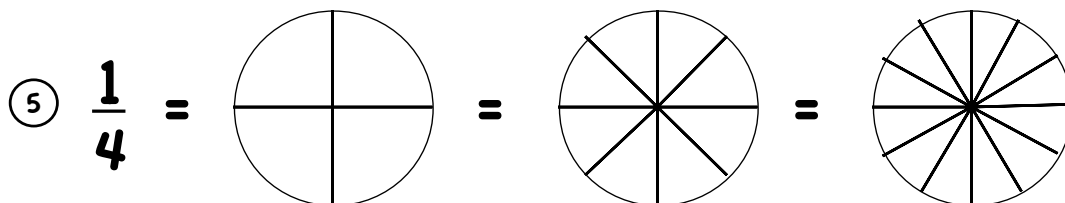
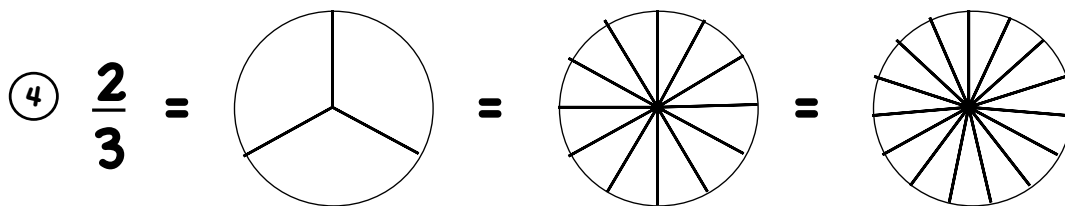
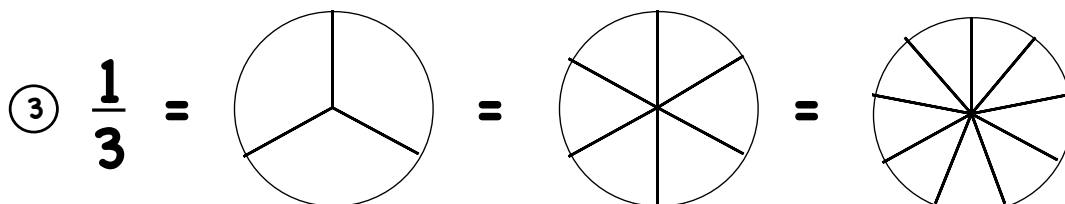
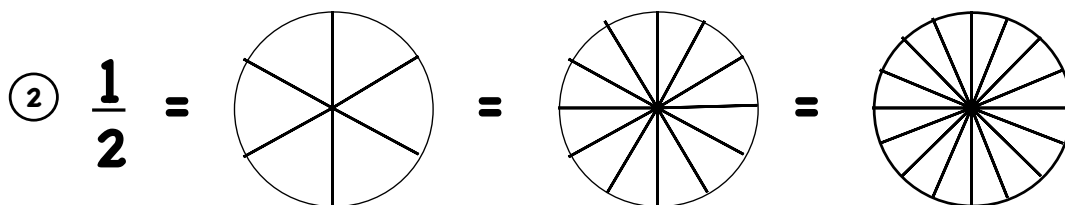
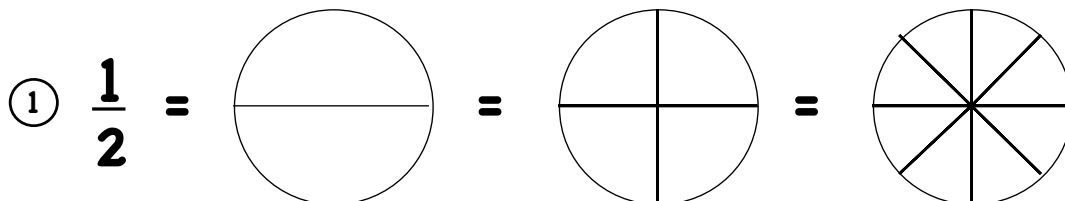
$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{2}{5}$	$\frac{4}{9}$
$\frac{9}{10}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{4}$



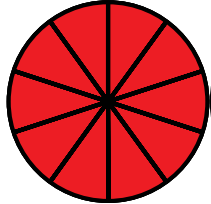
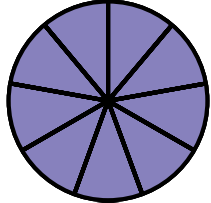
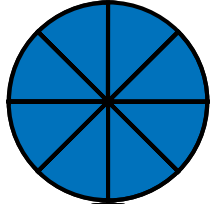
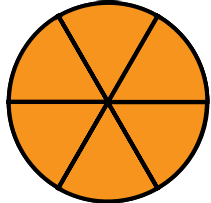
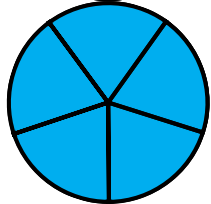
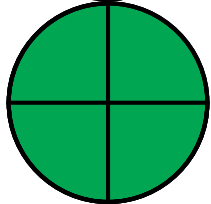
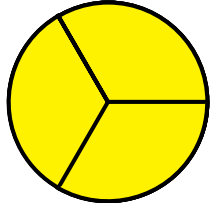
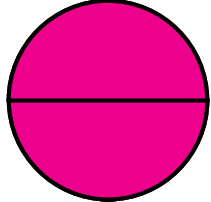
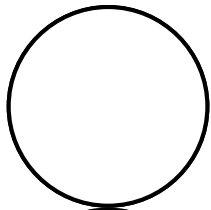


# Finding Equivalent Fractions with Fraction Circles

Show each fraction with Fraction Circles. Color the fraction pictures to match.



# Building Equivalent Fractions with Fraction Circles



For problems 1-10, use Fraction Circles to make as many fair trades as you can without mixing denominators. Think of a rule to work the starred problems without Fraction Circles.

$$\textcircled{1} \frac{1}{8} =$$

$$\textcircled{2} \frac{1}{6} =$$

$$\textcircled{3} \frac{5}{6} =$$

$$\textcircled{4} \frac{1}{5} =$$

$$\textcircled{5} \frac{4}{5} =$$

$$\textcircled{6} \frac{1}{4} =$$

$$\textcircled{7} \frac{3}{4} =$$

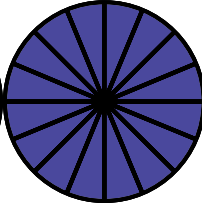
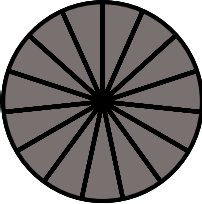
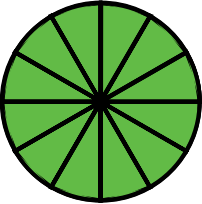
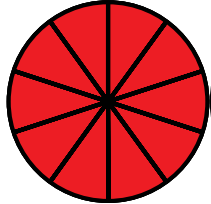
$$\textcircled{8} \frac{1}{3} =$$

$$\textcircled{9} \frac{2}{3} =$$

$$\textcircled{10} \frac{1}{2} =$$

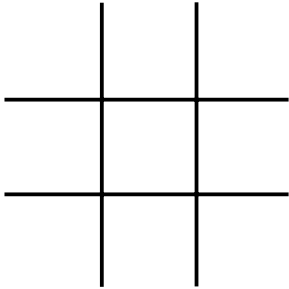
$$\star \frac{1}{9} =$$

$$\star \frac{9}{10} =$$

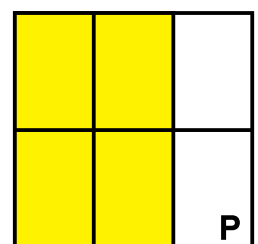
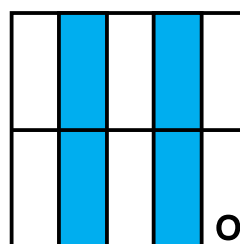
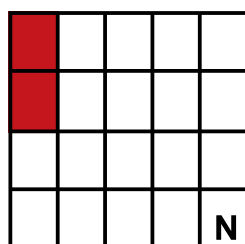
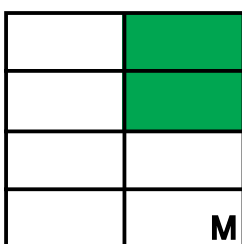
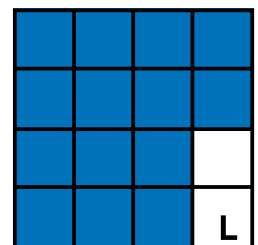
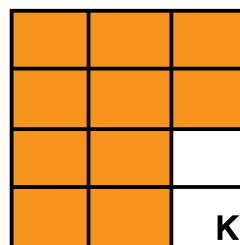
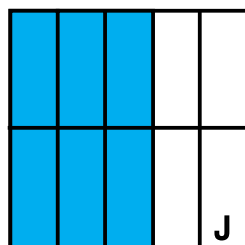
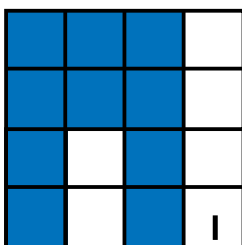
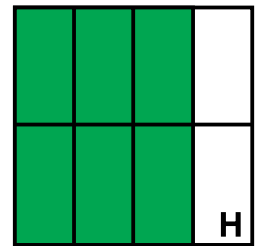
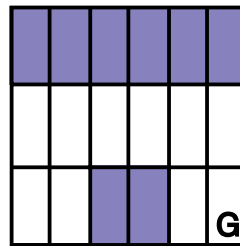
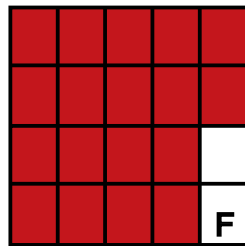
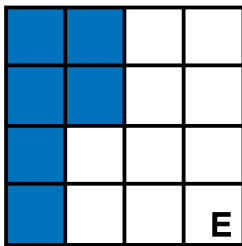
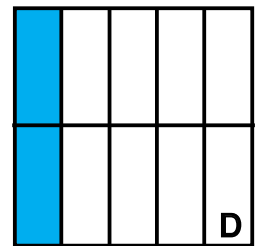
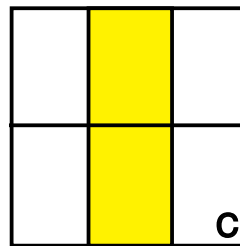
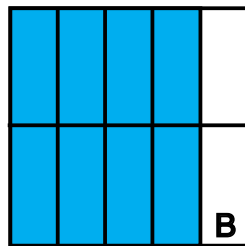
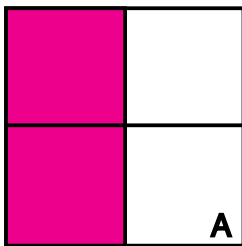


## Equivalent Fraction Tic-Tac-Toe

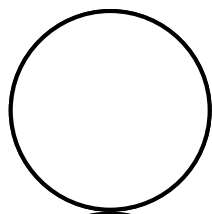
Fill in a tic-tac-toe grid with any nine of the fractions below. As the squares below are called by letter, cross out the fractions in the grid that correspond to how much of the squares are shaded. For example, if C is called, cross out  $\frac{1}{3}$  (the equivalent of  $\frac{2}{6}$ ). If E is called, cross out  $\frac{3}{8}$  (the equivalent of  $\frac{6}{16}$ ). The first to cross out three fractions in a row horizontally, vertically, or diagonally wins.



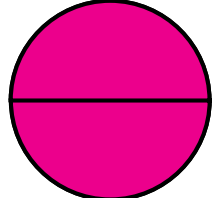
$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{2}{5}$	$\frac{4}{9}$
$\frac{9}{10}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{4}$



## Reducing Fractions with Fraction Circles



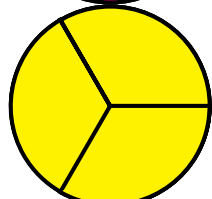
For problems 1-18, use Fraction Circles to find the fair trade that is in "lowest terms." Think of a rule to work the starred problems without Fraction Circles.



$$\textcircled{1} \quad \frac{4}{16} =$$

$$\textcircled{2} \quad \frac{2}{10} =$$

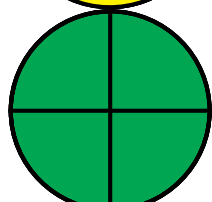
$$\textcircled{3} \quad \frac{3}{6} =$$



$$\textcircled{4} \quad \frac{2}{4} =$$

$$\textcircled{5} \quad \frac{8}{16} =$$

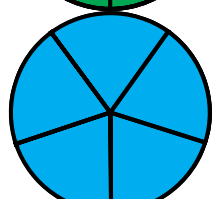
$$\textcircled{6} \quad \frac{5}{10} =$$



$$\textcircled{7} \quad \frac{3}{9} =$$

$$\textcircled{8} \quad \frac{6}{12} =$$

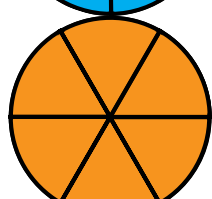
$$\textcircled{9} \quad \frac{5}{15} =$$



$$\textcircled{10} \quad \frac{2}{16} =$$

$$\textcircled{11} \quad \frac{3}{15} =$$

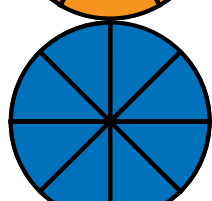
$$\textcircled{12} \quad \frac{6}{9} =$$



$$\textcircled{13} \quad \frac{9}{12} =$$

$$\textcircled{14} \quad \frac{4}{10} =$$

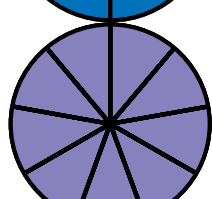
$$\textcircled{15} \quad \frac{4}{12} =$$



$$\textcircled{16} \quad \frac{2}{6} =$$

$$\textcircled{17} \quad \frac{8}{12} =$$

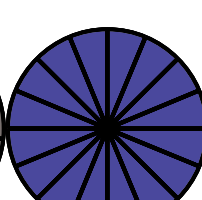
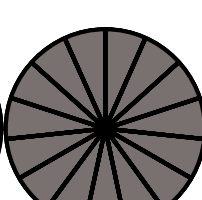
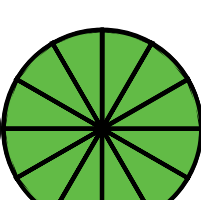
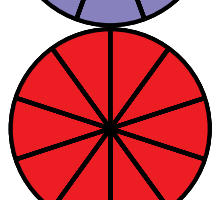
$$\textcircled{18} \quad \frac{12}{16} =$$



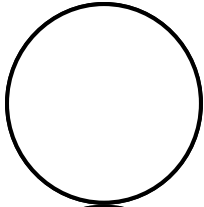
$$\star \quad \frac{4}{20} =$$

$$\star \quad \frac{20}{25} =$$

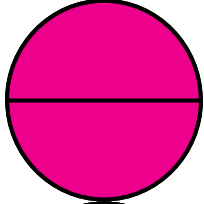
$$\star \quad \frac{50}{100} =$$



# Adding Fractions: Like Denominators



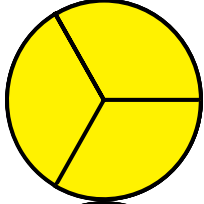
Work problems 1-18 with Fraction Circles. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.



①  $\frac{1}{3} + \frac{1}{3} =$

②  $\frac{2}{5} + \frac{2}{5} =$

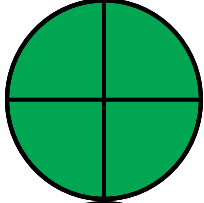
③  $\frac{3}{10} + \frac{3}{10} =$



④  $\frac{1}{6} + \frac{5}{6} =$

⑤  $\frac{2}{9} + \frac{7}{9} =$

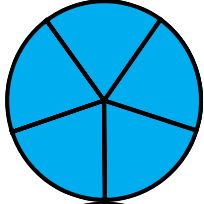
⑥  $\frac{7}{16} + \frac{9}{16} =$



⑦  $\frac{3}{4} + \frac{3}{4} =$

⑧  $\frac{5}{8} + \frac{7}{8} =$

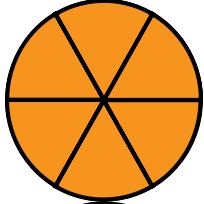
⑨  $\frac{7}{12} + \frac{11}{12} =$



⑩  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$

⑪  $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} =$

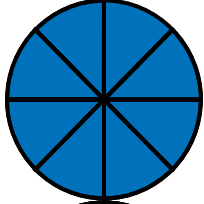
⑫  $\frac{1}{15} + \frac{4}{15} + \frac{7}{15} =$



⑬  $1\frac{1}{4} + 1\frac{1}{4} =$

⑭  $4\frac{3}{10} + 5\frac{7}{10} =$

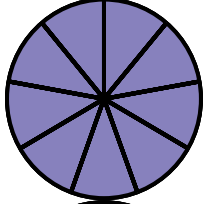
⑮  $\frac{7}{16} + 4\frac{11}{16} =$



⑯ 
$$\begin{array}{r} \frac{3}{5} \\ + \frac{4}{5} \\ \hline \end{array}$$

⑰ 
$$\begin{array}{r} \frac{5}{8} \\ + \frac{5}{8} \\ \hline \end{array}$$

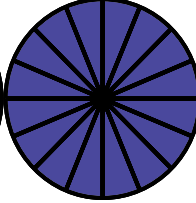
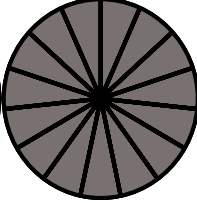
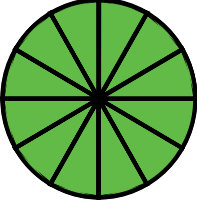
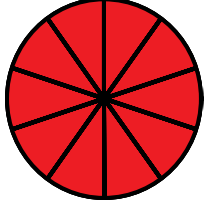
⑱ 
$$\begin{array}{r} \frac{5}{12} \\ + \frac{1}{12} \\ \hline \end{array}$$



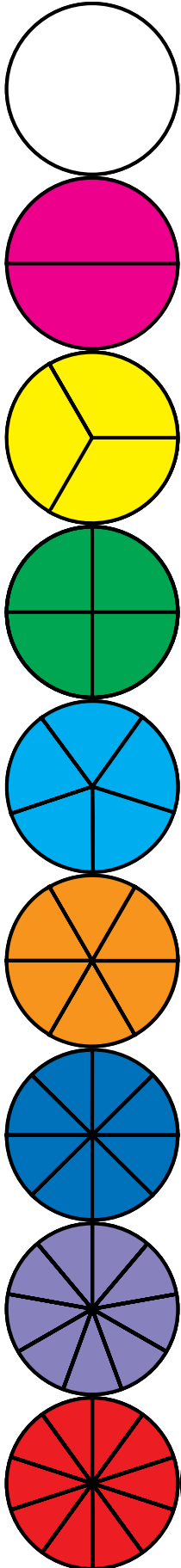
★  $\frac{3}{7} + \frac{4}{7} =$

★  $\frac{10}{11} + \frac{10}{11} =$

★  $\frac{7}{20} + \frac{9}{20} =$



# Adding Fractions: Unlike Denominators, One Color Change



Work problems 1-18 with Fraction Circles. For each problem, first make the colors the same. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.

①  $\frac{1}{2} + \frac{1}{4} =$       ②  $\frac{1}{4} + \frac{3}{8} =$       ③  $\frac{3}{5} + \frac{3}{10} =$

④  $\frac{1}{3} + \frac{1}{6} =$       ⑤  $\frac{1}{4} + \frac{5}{12} =$       ⑥  $\frac{2}{5} + \frac{4}{15} =$

⑦  $\frac{1}{2} + \frac{9}{10} =$       ⑧  $\frac{2}{3} + \frac{5}{6} =$       ⑨  $\frac{1}{3} + \frac{11}{12} =$

⑩  $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} =$       ⑪  $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} =$       ⑫  $\frac{1}{4} + \frac{1}{8} + \frac{3}{8} =$

⑬  $1\frac{1}{2} + 2\frac{1}{6} =$       ⑭  $3\frac{1}{3} + 4\frac{5}{9} =$       ⑮  $\frac{2}{3} + 4\frac{8}{15} =$

⑯ 
$$\begin{array}{r} \frac{1}{2} \\ + \frac{5}{12} \\ \hline \end{array}$$

⑰ 
$$\begin{array}{r} \frac{1}{2} \\ + \frac{9}{16} \\ \hline \end{array}$$

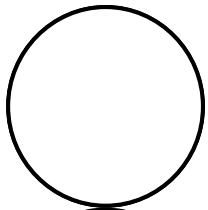
⑱ 
$$\begin{array}{r} \frac{1}{3} \\ + \frac{11}{12} \\ \hline \end{array}$$

★  $\frac{1}{9} + \frac{1}{18} =$

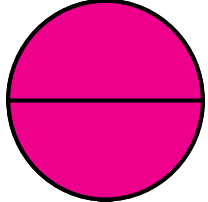
★  $\frac{7}{15} + \frac{11}{30} =$

★  $\frac{1}{2} + \frac{49}{50} =$

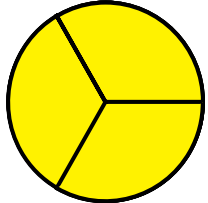
# Adding Fractions: Unlike Denominators, Two Color Changes



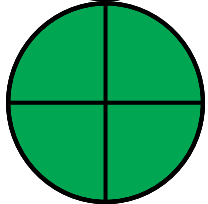
Work problems 1-18 with Fraction Circles. For each problem, first make the colors the same. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.



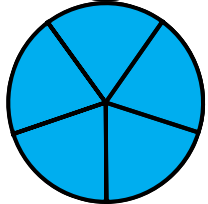
$$\textcircled{1} \quad \frac{1}{2} + \frac{1}{3} = \quad \textcircled{2} \quad \frac{1}{2} + \frac{1}{5} = \quad \textcircled{3} \quad \frac{1}{3} + \frac{1}{4} =$$



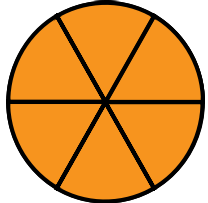
$$\textcircled{4} \quad \frac{1}{3} + \frac{1}{5} = \quad \textcircled{5} \quad \frac{1}{3} + \frac{2}{5} = \quad \textcircled{6} \quad \frac{1}{3} + \frac{3}{5} =$$



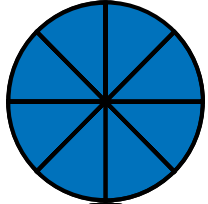
$$\textcircled{7} \quad \frac{1}{3} + \frac{3}{4} = \quad \textcircled{8} \quad \frac{2}{3} + \frac{3}{4} = \quad \textcircled{9} \quad \frac{2}{3} + \frac{4}{5} =$$



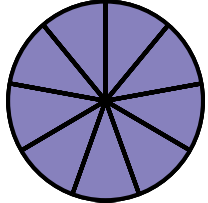
$$\textcircled{10} \quad \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = \quad \textcircled{11} \quad \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \quad \textcircled{12} \quad \frac{3}{4} + \frac{1}{6} + \frac{1}{6} =$$



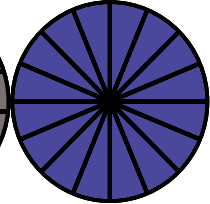
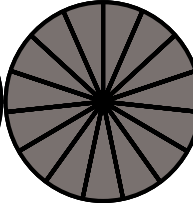
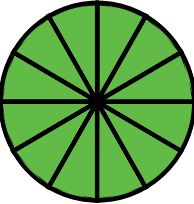
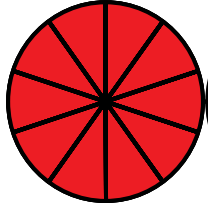
$$\textcircled{13} \quad 1\frac{1}{2} + 3\frac{2}{3} = \quad \textcircled{14} \quad 2\frac{1}{2} + 4\frac{4}{5} = \quad \textcircled{15} \quad \frac{1}{3} + 5\frac{4}{5} =$$



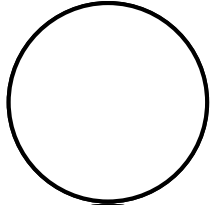
$$\textcircled{16} \quad \begin{array}{r} \frac{1}{2} \\ + \frac{3}{5} \\ \hline \end{array} \quad \textcircled{17} \quad \begin{array}{r} \frac{2}{3} \\ + \frac{1}{4} \\ \hline \end{array} \quad \textcircled{18} \quad \begin{array}{r} \frac{2}{3} \\ + \frac{4}{5} \\ \hline \end{array}$$



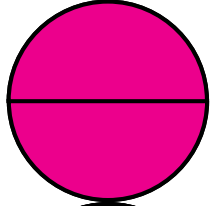
$$\star \frac{1}{2} + \frac{4}{9} = \quad \star \frac{2}{3} + \frac{3}{8} = \quad \star \frac{1}{4} + \frac{2}{5} =$$



# Subtracting Fractions: Like Denominators



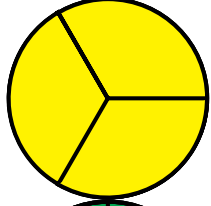
Work problems 1-18 with Fraction Circles. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.



①  $\frac{2}{3} - \frac{1}{3} =$

②  $\frac{5}{6} - \frac{1}{6} =$

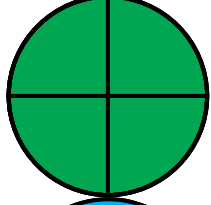
③  $\frac{3}{10} - \frac{1}{10} =$



④  $\frac{3}{4} - \frac{1}{4} =$

⑤  $\frac{3}{8} - \frac{1}{8} =$

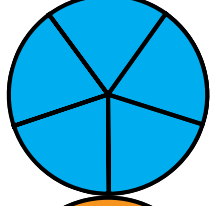
⑥  $\frac{5}{12} - \frac{1}{12} =$



⑦  $\frac{3}{5} - \frac{2}{5} =$

⑧  $\frac{4}{9} - \frac{2}{9} =$

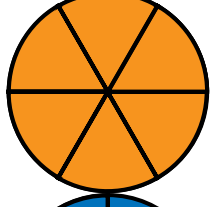
⑨  $\frac{4}{15} - \frac{2}{15} =$



⑩  $\frac{7}{8} - \frac{5}{8} =$

⑪  $\frac{11}{12} - \frac{11}{12} =$

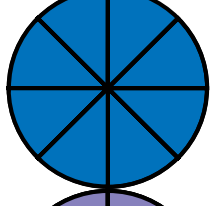
⑫  $\frac{15}{16} - \frac{11}{16} =$



⑬  $1\frac{3}{5} - 1\frac{1}{5} =$

⑭  $5\frac{1}{6} - 2\frac{1}{6} =$

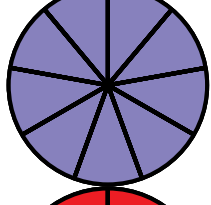
⑮  $1\frac{3}{8} - \frac{5}{8} =$



⑯ 
$$\begin{array}{r} \frac{4}{5} \\ - \frac{3}{5} \\ \hline \end{array}$$

⑰ 
$$\begin{array}{r} \frac{7}{8} \\ - \frac{3}{8} \\ \hline \end{array}$$

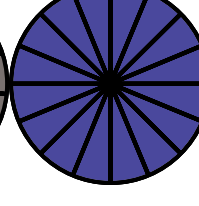
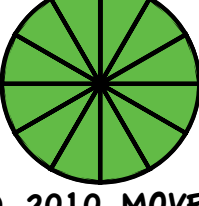
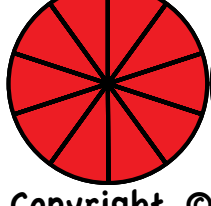
⑱ 
$$\begin{array}{r} 2\frac{7}{12} \\ - \frac{11}{12} \\ \hline \end{array}$$



★  $\frac{4}{7} - \frac{1}{7}$

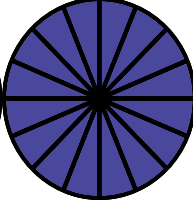
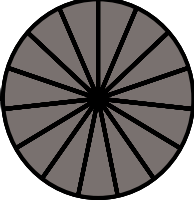
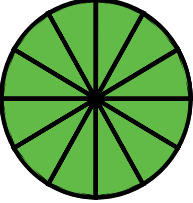
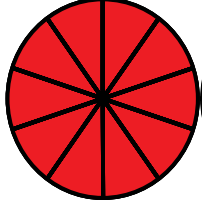
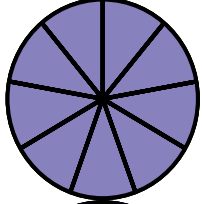
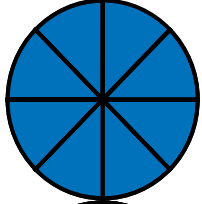
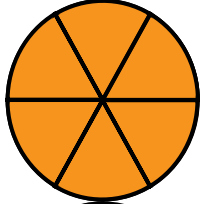
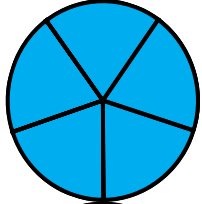
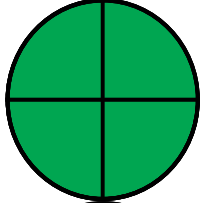
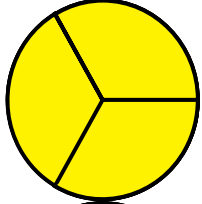
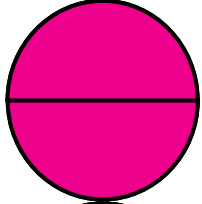
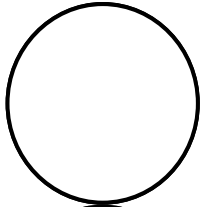
★  $\frac{10}{11} - \frac{9}{11} =$

★  $\frac{80}{99} - \frac{50}{99} =$





## Subtracting Fractions: Unlike Denominators, One Color Change



Work problems 1-18 with Fraction Circles. For each problem, first make the colors the same. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.

$$\textcircled{1} \quad \frac{1}{2} - \frac{1}{4} =$$

$$\textcircled{2} \quad \frac{1}{3} - \frac{1}{6} =$$

$$\textcircled{3} \quad \frac{3}{5} - \frac{3}{10} =$$

$$\textcircled{4} \quad \frac{3}{4} - \frac{3}{8} =$$

$$\textcircled{5} \quad \frac{5}{6} - \frac{7}{12} =$$

$$\textcircled{6} \quad \frac{2}{5} - \frac{4}{15} =$$

$$\textcircled{7} \quad \frac{9}{10} - \frac{1}{2} =$$

$$\textcircled{8} \quad \frac{5}{6} - \frac{1}{3} =$$

$$\textcircled{9} \quad \frac{3}{8} - \frac{1}{4} =$$

$$\textcircled{10} \quad \frac{11}{12} - \frac{1}{3} =$$

$$\textcircled{11} \quad \frac{5}{12} - \frac{1}{4} =$$

$$\textcircled{12} \quad \frac{9}{16} - \frac{3}{8} =$$

$$\textcircled{13} \quad 2\frac{1}{2} - 1\frac{1}{6} =$$

$$\textcircled{14} \quad 4\frac{5}{6} - 3\frac{2}{3} =$$

$$\textcircled{15} \quad 3\frac{1}{3} - 1\frac{4}{9} =$$

$$\textcircled{16} \quad \begin{array}{r} \frac{1}{2} \\ - \frac{5}{12} \\ \hline \end{array}$$

$$\textcircled{17} \quad \begin{array}{r} \frac{1}{2} \\ - \frac{5}{16} \\ \hline \end{array}$$

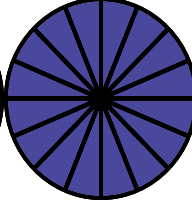
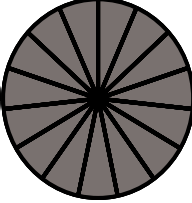
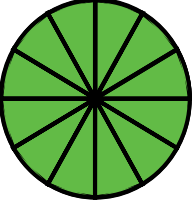
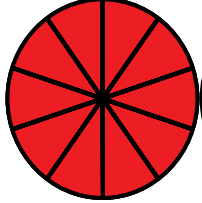
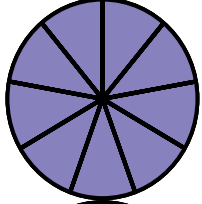
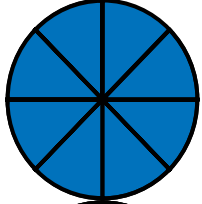
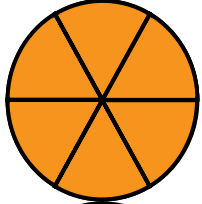
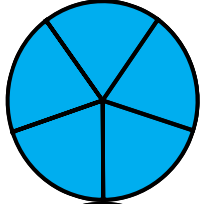
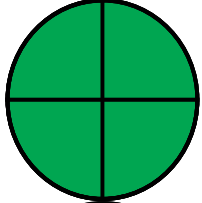
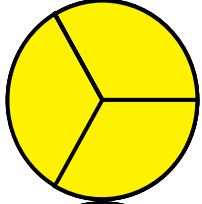
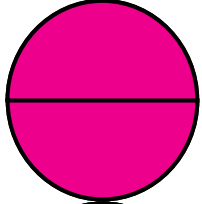
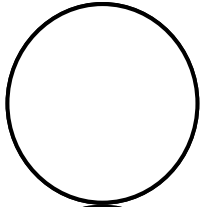
$$\textcircled{18} \quad \begin{array}{r} 2\frac{1}{3} \\ - \frac{5}{12} \\ \hline \end{array}$$

$$\star \quad \frac{1}{9} - \frac{1}{18} =$$

$$\star \quad \frac{7}{20} - \frac{3}{10} =$$

$$\star \quad 2 - \frac{24}{25} =$$

## Subtracting Fractions: Unlike Denominators, Two Color Changes



Work problems 1-18 with Fraction Circles. For each problem, first make the colors the same. Reduce your answers if you can. Think of a rule to work the starred problems without Fraction Circles.

$$\textcircled{1} \quad \frac{1}{2} - \frac{1}{3} =$$

$$\textcircled{2} \quad \frac{1}{2} - \frac{1}{5} =$$

$$\textcircled{3} \quad \frac{1}{3} - \frac{1}{4} =$$

$$\textcircled{4} \quad \frac{1}{3} - \frac{1}{5} =$$

$$\textcircled{5} \quad \frac{2}{3} - \frac{2}{5} =$$

$$\textcircled{6} \quad \frac{2}{3} - \frac{3}{5} =$$

$$\textcircled{7} \quad \frac{3}{4} - \frac{1}{3} =$$

$$\textcircled{8} \quad \frac{3}{4} - \frac{2}{3} =$$

$$\textcircled{9} \quad \frac{4}{5} - \frac{1}{3} =$$

$$\textcircled{10} \quad \frac{4}{5} - \frac{2}{3} =$$

$$\textcircled{11} \quad \frac{2}{3} - \frac{1}{2} =$$

$$\textcircled{12} \quad \frac{4}{5} - \frac{1}{2} =$$

$$\textcircled{13} \quad 3\frac{1}{4} - 3\frac{1}{6} =$$

$$\textcircled{14} \quad 4\frac{5}{6} - 2\frac{3}{4} =$$

$$\textcircled{15} \quad 3\frac{1}{2} - 2\frac{2}{3} =$$

$$\textcircled{16} \quad \begin{array}{r} \frac{1}{2} \\ - \frac{2}{5} \\ \hline \end{array}$$

$$\textcircled{17} \quad \begin{array}{r} \frac{2}{3} \\ - \frac{1}{4} \\ \hline \end{array}$$

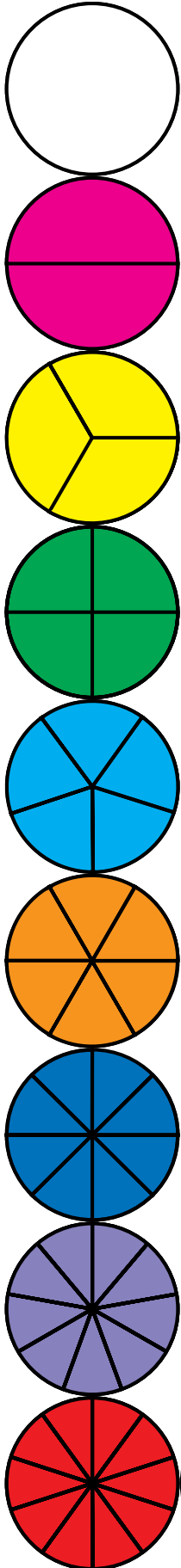
$$\textcircled{18} \quad \begin{array}{r} 1\frac{2}{3} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$\star \quad \frac{1}{2} - \frac{1}{9} =$$

$$\star \quad \frac{4}{5} - \frac{3}{4} =$$

$$\star \quad 2\frac{1}{3} - \frac{5}{8} =$$

# Multiplying Fractions with Fraction Circles



Work problems 1-18 with Fraction Circles. Think of the times sign as the word "of." For example,  $\frac{1}{2} \times \frac{1}{4}$  means  $\frac{1}{2}$  "of"  $\frac{1}{4}$ . Also, since  $\frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2}$ ,  $\frac{1}{2}$  of  $\frac{1}{4}$  means the same as  $\frac{1}{4}$  of  $\frac{1}{2}$ , so the problems can be worked from either the left or the right. Think of a rule to work the starred problems without Fraction Circles.

$$\textcircled{1} \quad \frac{1}{2} \times \frac{1}{4} = \quad \textcircled{2} \quad \frac{1}{2} \times \frac{1}{2} = \quad \textcircled{3} \quad \frac{1}{2} \times \frac{2}{3} =$$

$$\textcircled{4} \quad \frac{1}{3} \times \frac{1}{2} = \quad \textcircled{5} \quad \frac{1}{3} \times \frac{1}{3} = \quad \textcircled{6} \quad \frac{1}{3} \times \frac{3}{4} =$$

$$\textcircled{7} \quad \frac{2}{3} \times \frac{1}{2} = \quad \textcircled{8} \quad \frac{2}{3} \times \frac{1}{3} = \quad \textcircled{9} \quad \frac{2}{3} \times \frac{3}{5} =$$

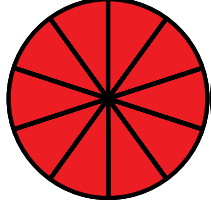
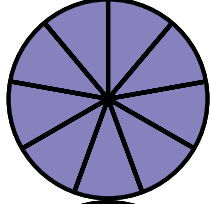
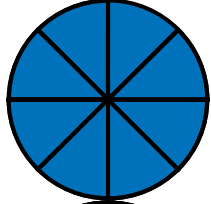
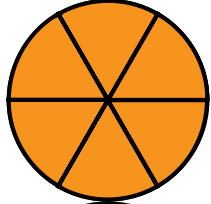
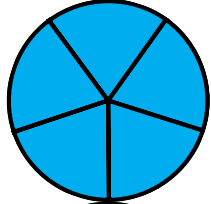
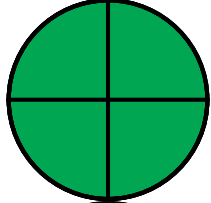
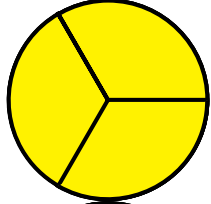
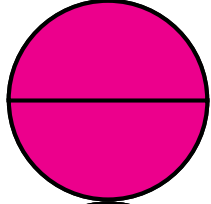
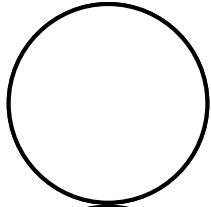
$$\textcircled{10} \quad \frac{1}{4} \times \frac{1}{2} = \quad \textcircled{11} \quad \frac{1}{4} \times \frac{1}{4} = \quad \textcircled{12} \quad \frac{1}{4} \times \frac{4}{5} =$$

$$\textcircled{13} \quad \frac{3}{4} \times \frac{1}{2} = \quad \textcircled{14} \quad \frac{3}{4} \times \frac{1}{3} = \quad \textcircled{15} \quad \frac{3}{4} \times \frac{2}{3} =$$

$$\textcircled{16} \quad \frac{1}{5} \times \frac{1}{3} = \quad \textcircled{17} \quad \frac{1}{6} \times \frac{1}{2} = \quad \textcircled{18} \quad \frac{7}{8} \times \frac{1}{2} =$$

$$\star \frac{1}{2} \times \frac{1}{10} = \quad \star \frac{1}{4} \times \frac{1}{5} = \quad \star \frac{1}{10} \times \frac{10}{20} =$$

# Dividing Fractions with Fraction Circles



Work problems 1-18 with Fraction Circles. The division sign asks how many of the second fraction are in the first one. For example,  $\frac{1}{2} \div \frac{1}{4}$  asks how many fourths in a half (two), just like  $10 \div 2$  asks how many 2s in 10 (five). Think of a rule to work the starred problems without Fraction Circles.

$$\textcircled{1} \quad \frac{1}{2} \div \frac{1}{6} =$$

$$\textcircled{2} \quad \frac{1}{2} \div \frac{1}{10} =$$

$$\textcircled{3} \quad \frac{1}{3} \div \frac{1}{9} =$$

$$\textcircled{4} \quad \frac{1}{3} \div \frac{1}{12} =$$

$$\textcircled{5} \quad \frac{1}{5} \div \frac{1}{10} =$$

$$\textcircled{6} \quad \frac{1}{5} \div \frac{1}{15} =$$

$$\textcircled{7} \quad \frac{2}{3} \div \frac{1}{9} =$$

$$\textcircled{8} \quad \frac{3}{4} \div \frac{1}{8} =$$

$$\textcircled{9} \quad \frac{3}{4} \div \frac{1}{12} =$$

$$\textcircled{10} \quad \frac{2}{5} \div \frac{1}{15} =$$

$$\textcircled{11} \quad \frac{3}{5} \div \frac{1}{10} =$$

$$\textcircled{12} \quad \frac{5}{6} \div \frac{1}{12} =$$

$$\textcircled{13} \quad \frac{2}{3} \div \frac{2}{9} =$$

$$\textcircled{14} \quad \frac{2}{3} \div \frac{2}{15} =$$

$$\textcircled{15} \quad \frac{3}{4} \div \frac{3}{8} =$$

$$\textcircled{16} \quad \frac{3}{4} \div \frac{3}{16} =$$

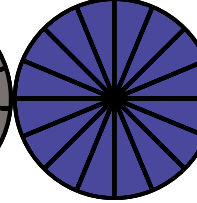
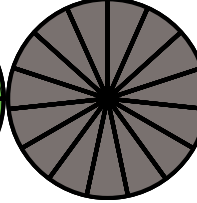
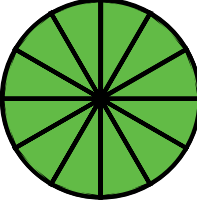
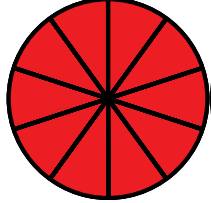
$$\textcircled{17} \quad \frac{3}{5} \div \frac{3}{10} =$$

$$\textcircled{18} \quad \frac{4}{5} \div \frac{2}{15} =$$

$$\star \quad \frac{1}{2} \div \frac{1}{20} =$$

$$\star \quad \frac{2}{3} \div \frac{1}{18} =$$

$$\star \quad \frac{3}{4} \div \frac{7}{28} =$$



1. < 2. < 3. < 4. > 5. > 6. > 7. = 8. = 9. = 10. = 11. = 12. =  
 13. < 14. < 15. < 16. < 17. < 18. < 19. > 20. > 21. > 22. < 23. > 24. =

## Page 4

1.  $1/4 < 1/3 < 1/2$       2.  $1/4 < 1/2 < 7/12$       3.  $2/9 < 2/5 < 2/3$       4.  $7/16 < 1/2 < 5/8$   
 5.  $3/8 < 3/5 < 3/4$       6.  $4/15 < 1/3 < 5/12$       7.  $1/5 < 2/5 < 3/5$       8.  $5/9 < 2/3 < 11/15$   
 9.  $1/8 < 3/8 < 5/8$       10.  $3/4 < 13/16 < 7/8$       11.  $2/9 < 5/9 < 8/9$       12.  $7/10 < 11/15 < 4/5$

## Page 7

1.  $2/16$       2.  $2/12$       3.  $10/12$       4.  $2/10 = 3/15$   
 5.  $8/10$       6.  $2/8 = 3/12 = 4/16$       7.  $6/8 = 9/12 = 12/16$       8.  $2/6 = 3/9 = 4/12 = 5/15$   
 9.  $4/6 = 6/9 = 8/12 = 10/15$       10.  $2/4 = 3/6 = 4/8 = 5/10 = 6/12 = [7/14] = 8/16$   
 Star:  $1/9 = 2/18 = 3/27 = \dots$       Star:  $9/10 = 18/20 = 27/30 = \dots$

## Page 9

1.  $1/4$       2.  $1/5$       3.  $1/2$       4.  $1/2$       5.  $1/2$       6.  $1/2$   
 7.  $1/3$       8.  $1/2$       9.  $1/3$       10.  $1/8$       11.  $1/5$       12.  $2/3$   
 13.  $3/4$       14.  $2/5$       15.  $1/3$       16.  $1/3$       17.  $2/3$       18.  $3/4$   
 Star:  $4/20 = 1/5$       Star:  $20/25 = 4/5$       Star:  $50/100 = 1/2$

## Page 10

1.  $2/6 = 1/3$       2.  $4/5$       3.  $6/10 = 3/5$   
 4.  $6/6 = 1$       5.  $9/9 = 1$       6.  $16/16 = 1$   
 7.  $6/4 = 1-1/2$       8.  $12/8 = 1-4/8 = 1-1/2$       9.  $18/12 = 1-6/12 = 1-1/2$   
 10.  $3/2 = 1-1/2$       11.  $6/9 = 2/3$       12.  $12/15 = 4/5$   
 13.  $2-2/4 = 2-1/2$       14.  $9-10/10 = 10$       15.  $4-18/16 = 5-2/16 = 5-1/8$   
 16.  $7/5 = 1-2/5$       17.  $10/8 = 1-2/8 = 1-1/4$       18.  $6/12 = 1/2$   
 Star:  $7/7 = 1$       Star:  $20/11 = 1-9/11$       Star:  $16/20 = 4/5$

1.  $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

3.  $\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$

5.  $\frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$

7.  $\frac{5}{10} + \frac{9}{10} = \frac{14}{10} = 1 - \frac{4}{10} = 1 - \frac{2}{5}$

9.  $\frac{4}{12} + \frac{11}{12} = \frac{15}{12} = 1 - \frac{3}{12} = 1 - \frac{1}{4}$

11.  $\frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

13.  $1 - \frac{3}{6} + 2 - \frac{1}{6} = 3 - \frac{4}{6} = 3 - \frac{2}{3}$

15.  $\frac{10}{15} + 4 - \frac{8}{15} = 4 - \frac{18}{15} = 5 - \frac{3}{15} = 5 - \frac{1}{5}$

17.  $\frac{8}{16} + \frac{9}{16} = \frac{17}{16} = 1 - \frac{1}{16}$

Star:  $\frac{2}{18} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$

Star:  $\frac{25}{50} + \frac{49}{50} = \frac{74}{50} = 1 - \frac{24}{50} = 1 - \frac{12}{25}$

## Page 12

1.  $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

3.  $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

5.  $\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

7.  $\frac{4}{12} + \frac{9}{12} = \frac{13}{12} = 1 - \frac{1}{12}$

9.  $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1 - \frac{7}{15}$

11.  $\frac{4}{12} + \frac{3}{12} + \frac{3}{12} = \frac{10}{12} = \frac{5}{6}$

13.  $4 - \frac{3}{6} + \frac{4}{6} = 4 - \frac{7}{6} = 5 - \frac{1}{6}$

15.  $\frac{5}{15} + 5 - \frac{12}{15} = 5 - \frac{17}{15} = 6 - \frac{2}{15}$

17.  $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

Star:  $\frac{9}{18} + \frac{8}{18} = \frac{17}{18}$

Star:  $\frac{5}{20} + \frac{8}{20} = \frac{13}{20}$

2.  $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

4.  $\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

6.  $\frac{6}{15} + \frac{4}{15} = \frac{10}{15} = \frac{2}{3}$

8.  $\frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1 - \frac{3}{6} = 1 - \frac{1}{2}$

10.  $\frac{2}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

12.  $\frac{2}{8} + \frac{1}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

14.  $3 - \frac{3}{9} + 4 - \frac{5}{9} = 7 - \frac{8}{9}$

16.  $\frac{6}{12} + \frac{5}{12} = \frac{11}{12}$

18.  $\frac{4}{12} + \frac{11}{12} = \frac{15}{12} = 1 - \frac{3}{12} = 1 - \frac{1}{4}$

Star:  $\frac{14}{30} + \frac{11}{30} = \frac{25}{30} = \frac{5}{6}$

2.  $\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

4.  $\frac{5}{15} + \frac{3}{15} = \frac{8}{15}$

6.  $\frac{5}{15} + \frac{9}{15} = \frac{14}{15}$

8.  $\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1 - \frac{5}{12}$

10.  $1 - \frac{1}{3} + \frac{3}{6} + \frac{3}{6} + \frac{2}{6} = \frac{8}{6} = 1 - \frac{2}{6} = 1 - \frac{1}{3}$

12.  $\frac{9}{12} + \frac{2}{12} + \frac{2}{12} = \frac{13}{12} = 1 - \frac{1}{12}$

14.  $6 - \frac{5}{10} + \frac{8}{10} = 6 - \frac{13}{10} = 7 - \frac{3}{10}$

16.  $\frac{5}{10} + \frac{6}{10} = \frac{11}{10} = 1 - \frac{1}{10}$

18.  $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1 - \frac{7}{15}$

Star:  $\frac{16}{24} + \frac{9}{24} = \frac{25}{24} = 1 - \frac{1}{24}$

**A**  
1.  $1/3$   
5.  $2/8 = 1/4$   
9.  $2/15$

2.  $4/6 = 2/3$   
6.  $4/12 = 1/3$   
10.  $2/8 = 1/4$

3.  $2/10 = 1/5$   
7.  $1/5$   
11. 0

4.  $2/4 = 1/2$   
8.  $2/9$   
12.  $4/16 = 1/4$

13.  $2/5$   
17.  $4/8 = 1/2$   
Star:  $3/7$

**N**

14. 3  
18.  $1 - 19/12 - 11/12 = 1 - 8/12 = 1 - 2/3$   
Star:  $1/11$

15.  $11/8 - 5/8 = 6/8 = 3/4$   
16.  $1/5$   
Star:  $30/99$

1.  $1/4$   
5.  $3/12 = 1/4$   
9.  $1/8$

**S**  
2.  $1/6$   
6.  $2/15$   
10.  $7/12$

3.  $3/10$   
7.  $4/10 = 2/5$   
11.  $2/12 = 1/6$

4.  $3/8$   
8.  $3/6 = 1/2$   
12.  $3/16$

13.  $1 - 2/6 = 1 - 1/3$   
17.  $3/16$   
Star:  $1/9$

14.  $1 - 1/6$   
18.  $1 - 7/12$   
Star:  $1/20$

**W**  
Star:  $1 - 1/25$

1.  $1/6$   
5.  $4/15$   
9.  $7/15$

2.  $3/10$   
6.  $1/15$   
10.  $2/15$

**E**  
3.  $1/12$   
7.  $5/12$   
11.  $1/6$

4.  $2/15$   
8.  $1/12$   
12.  $3/10$

13.  $2/24 = 1/12$   
17.  $5/12$   
Star:  $7/18$

14.  $2 - 1/12$   
18.  $11/12$   
Star:  $1/20$

15.  $5/6$   
Star:  $1 - 17/24$

**R**

**S**

**A**  
1.  $1/8$   
5.  $1/9$   
9.  $6/15 = 2/5$

2.  $1/4$   
6.  $3/12 = 1/4$   
10.  $1/8$

3.  $2/6 = 1/3$   
7.  $2/6 = 1/3$   
11.  $1/16$

4.  $1/6$   
8.  $2/9$   
12.  $4/20 = 1/5$

13.  $3/8$

14.  $3/12 = 1/4$

15.  $6/12 = 1/2$

16.  $1/15$

17.  $1/12$

18.  $7/16$

Star:  $1/20$

Star:  $1/20$

Star:  $1/20$

**N**

Page 17

1. 3    2. 5    3. 3    4. 4    5. 2    6. 3    7. 6    8. 6    9. 9    10. 6    11. 6    12. 10

13. 3    14. 5    15. 2    16. 4    17. 2    18. 6    Star: 10    Star: 12    Star: 3

**S**

**W**

**E**

**R**

**S**